Statistics Refresher

Day 3

Recap

- Statistics is about summarizing and analyzing data
- Data consist of a set of *variables* about some number of *cases*

Recap

- Three measures of central tendancy
  - Mean $$\bar{y} = \frac{\sum y}{n}$$
  - Median - middle number when cases listed in order
  - Mode - observation that appears the most

Calculate the mean, median, and mode

3	0	4	9	5	8	2	6	3	9

$$\bar{x} = \frac{3+0+4+9+5+8+2+6+3+9}{10}$$

$$= \frac{4.9}{10} = 4.9$$

0	2	3	3	4	5	6	8	9	9

$$\frac{4+5}{2} = 4.5$$

Mode: 3, 9

Recap

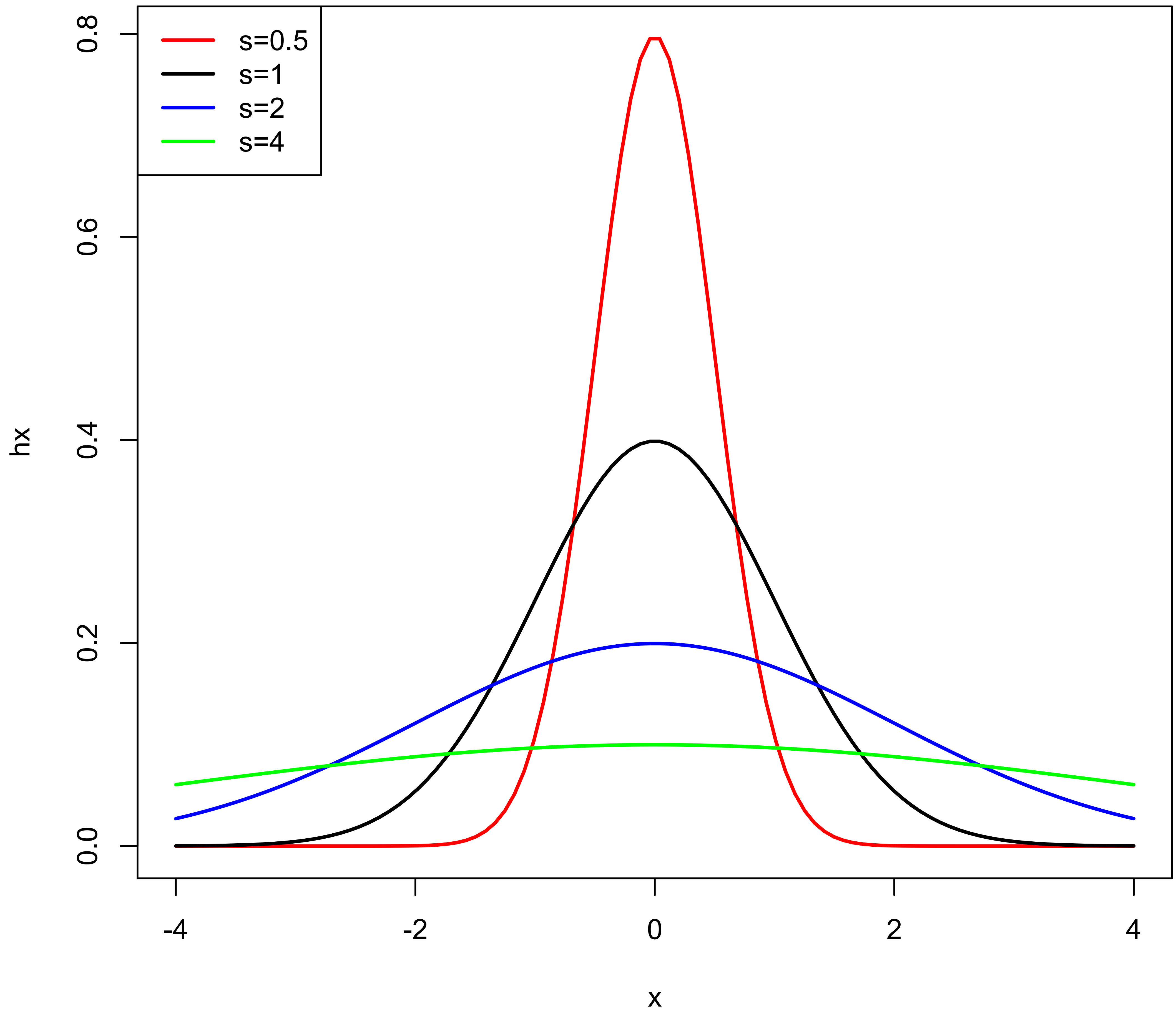
- Variance and Standard Deviation tell us how spread out the data are

Variance

$$s^2 = \frac{\sum (y_i - \bar{y})^2}{n - 1}$$

Standard Deviation

$$s = \sqrt{s^2} = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n - 1}}$$


s=0.5
s=1
s=2
s=4
0.0
0.2
0.4
0.6
0.8
hx
-4
-2
0
2
4
x

Recap

- Inferential statistics lets us make inferences about a population based on a sample
- Populations have parameters, samples have statistics

Recap

- Lots of possible samples from one population
- The distribution of sample statistics from all the possible samples of a population is called the sampling distribution

Recap

Sampling Distribution Mean

$$\mu_{\bar{X}} = \frac{\sum \bar{X}}{N}$$

Standard Error

$$\sigma_{\bar{X}} = \sqrt{\frac{\sum(X - \mu_{\bar{X}})^2}{N}}$$

Recap

- Z scores are standardized deviation scores

$$Z = \frac{X_i - \bar{X}}{s}$$

What is the Z score for the value of 4 in a sample with mean 5 and standard deviation 0.7?

$$Z = \frac{X_i - \bar{X}}{s}$$
$$= \frac{4-5}{0.7}$$
$$= -1.429$$

Assuming the sample if normally distributed, what is the proportion of the sample with values below 4? Above 4?

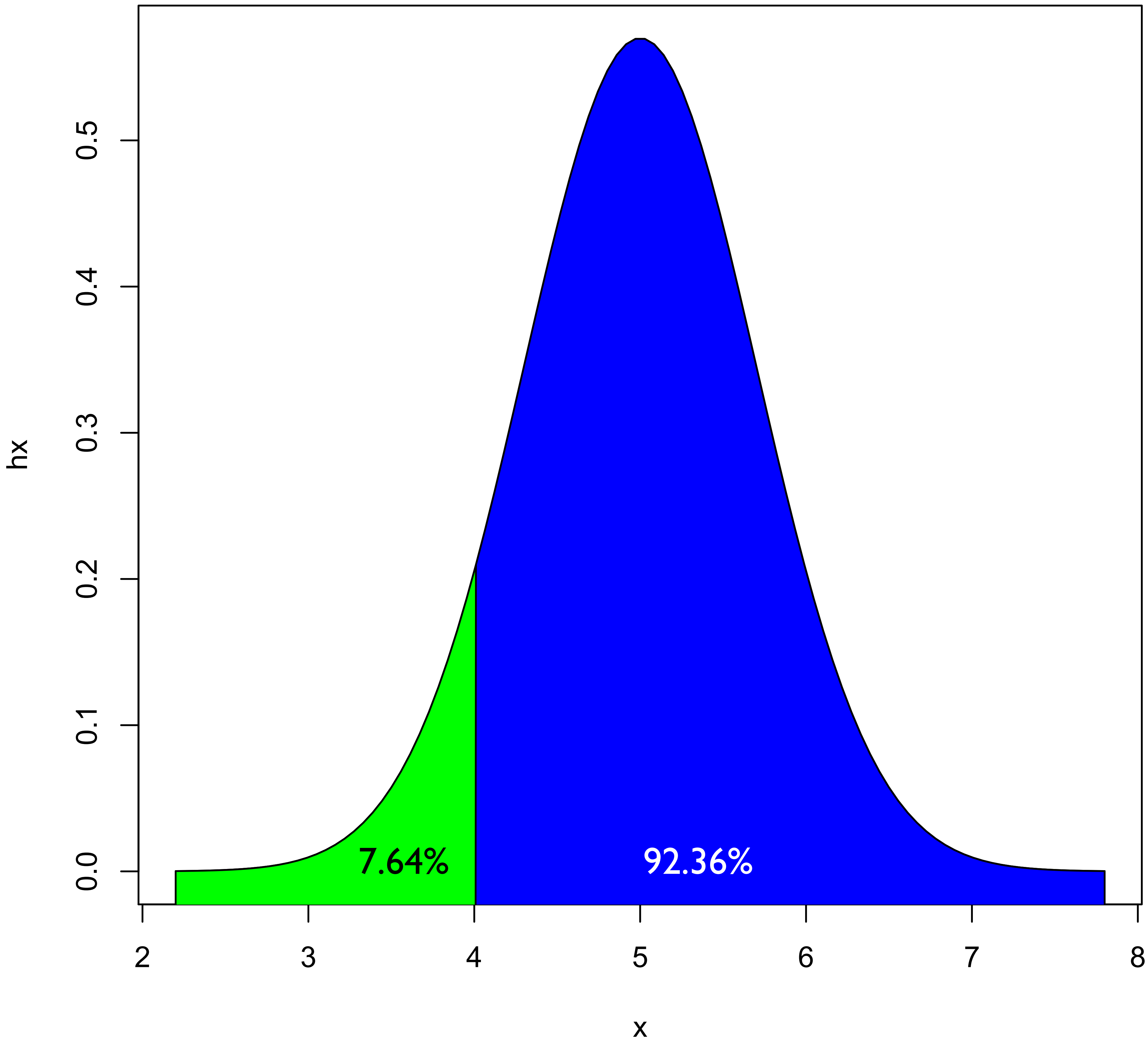
$$Z = -1.429$$

Second decimal place in z										
0.09	*0.08*	*0.07*	*0.06*	*0.05*	*0.04*	*0.03*	*0.02*	*0.01*	*0.00*	*z*
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	*−1.4*
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	*−1.3*
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	*−1.2*
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	*−1.1*
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	*−1.0*

So 7.64% of the sample has values below 4

$$1 - 0.0764 = 0.9236$$

And 92.36% of the sample has values above 4


hx
0.0
0.1
0.2
0.3
0.4
0.5
2
3
4
5
6
7
8
x
7.64%
92.36%

Recap

- We can use the sampling distribution to construct confidence intervals for population means and proportions

$$\bar{X} \pm Z(\sigma_{\bar{X}})$$

$$\bar{X} \pm t(\sigma_{\bar{X}})$$

$$\hat{\pi} \pm Z\sigma_{\hat{\pi}}$$

Sample Problem

- You have a sample of 50 iPhones and find that the average number of third party apps installed is 12 apps, with a standard deviation of 2. Construct a 95% confidence interval for the population mean number of apps installed.

$$\bar{x} = 12 \quad s = 2 \quad n = 50$$

$$\bar{x} \pm Z\left(\frac{s}{\sqrt{n}}\right)$$

$$12 \pm 1.96\left(\frac{2}{\sqrt{50}}\right)$$

$$12 \pm 1.96(0.283)$$

$$12 \pm 0.554$$

We are 95% confident that the population mean number of third party apps installed on iPhones is between 11.446 and 12.554 apps.

Sample Problem

- In the same sample of iPhones (n=50), you find that 15 of them are the latest generation, iPhone 5. Construct a 95% confidence interval for the population proportion of iPhone 5.

$$x = 15 \quad n = 50$$

$$\hat{\pi} \pm Z\left(\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}}\right) \qquad \hat{\pi} = \frac{15}{50} = 0.3$$

$$0.3 \pm 1.96\left(\sqrt{\frac{0.3(1-0.3)}{50}}\right)$$

$$0.3 \pm 1.96\left(\sqrt{\frac{.21}{50}}\right)$$

$$0.3 \pm 1.96(0.0648)$$

$$0.3 \pm 0.127$$

We can be 95% confident that the population proportion of iPhone 5 is between 0.173 and 0.427

Sample Problem

- You find a sample of 10 android phones and find that the average number of third party apps installed is 16 apps with a standard deviation of 5. Construct a 95% confidence interval for the population mean number of apps installed on android phones.

$$\bar{x} = 16 \quad s = 5 \quad n = 10$$

$$\bar{x} \pm t\left(\frac{s}{\sqrt{n}}\right) \qquad t = 2.262$$

$$16 \pm 2.262\left(\frac{5}{\sqrt{10}}\right)$$

$$16 \pm 2.262(1.581)$$

$$16 \pm 3.577$$

We can be 95% confident that the population mean number of third party apps installed on android phones is between 12.423 and 19.577

Hypothesis Testing

Hypothesis Testing

- We can use the methods we've been talking about to test whether a hypothesis is true
- We will start with testing values of a mean

Steps

1. State the assumptions
2. State the hypothesis
3. Compute the test statistic
4. Find the p-value/critical region
5. State the decision and conclusion

1. Assumptions

- Data are independently random
- For tests of the mean with large samples:
  - Normal sampling distribution
- For small samples:
  - Population is normally distributed

Assumptions

- Before proceeding, consider whether there is any evidence that any of the assumptions are being violated
- Don't need to prove the assumptions, but should pause to think before moving on

2. Hypotheses

- Construct two hypotheses:
  - Null Hypothesis H_0
  - Alternative Hypothesis H_A or H_1

Null Hypothesis

- Typically has the form: $H_0 : \mu = \mu_0$
- Think of this as no effect or no change compared to some standard

Alternative Hypothesis

- For two-tailed test, typically has the form:

$$H_A : \mu \neq \mu_0$$

- For one-tailed test:

$$H_A : \mu > \mu_0 \qquad H_A : \mu < \mu_0$$

- This is your research hypotheses, that something has an effect or is different than normal

One vs Two Tailed

- Tests can either be one tailed or two tailed
- One tailed when you have an expectation about the direction of effect
- Two tailed when you have no expectation about the direction of effect
- One tailed tests are more powerful

Errors

- Two types of errors you can make:
  - Type I - rejecting a true null hypotheses
  - Type II - failing to reject a false null hypotheses

Significance Level

- This is the probability of making a type I error
- Decided before doing the test
- Denoted by alpha (α)
- Common alphas: 0.01, 0.05, 0.001

Type II Error

- Denoted by beta (β)
- Indirectly proportion to alpha - as alpha increases, beta decreases and vice versa
- So you don't want to choose an alpha that's too small

3. Compute Test Statistic

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}$$

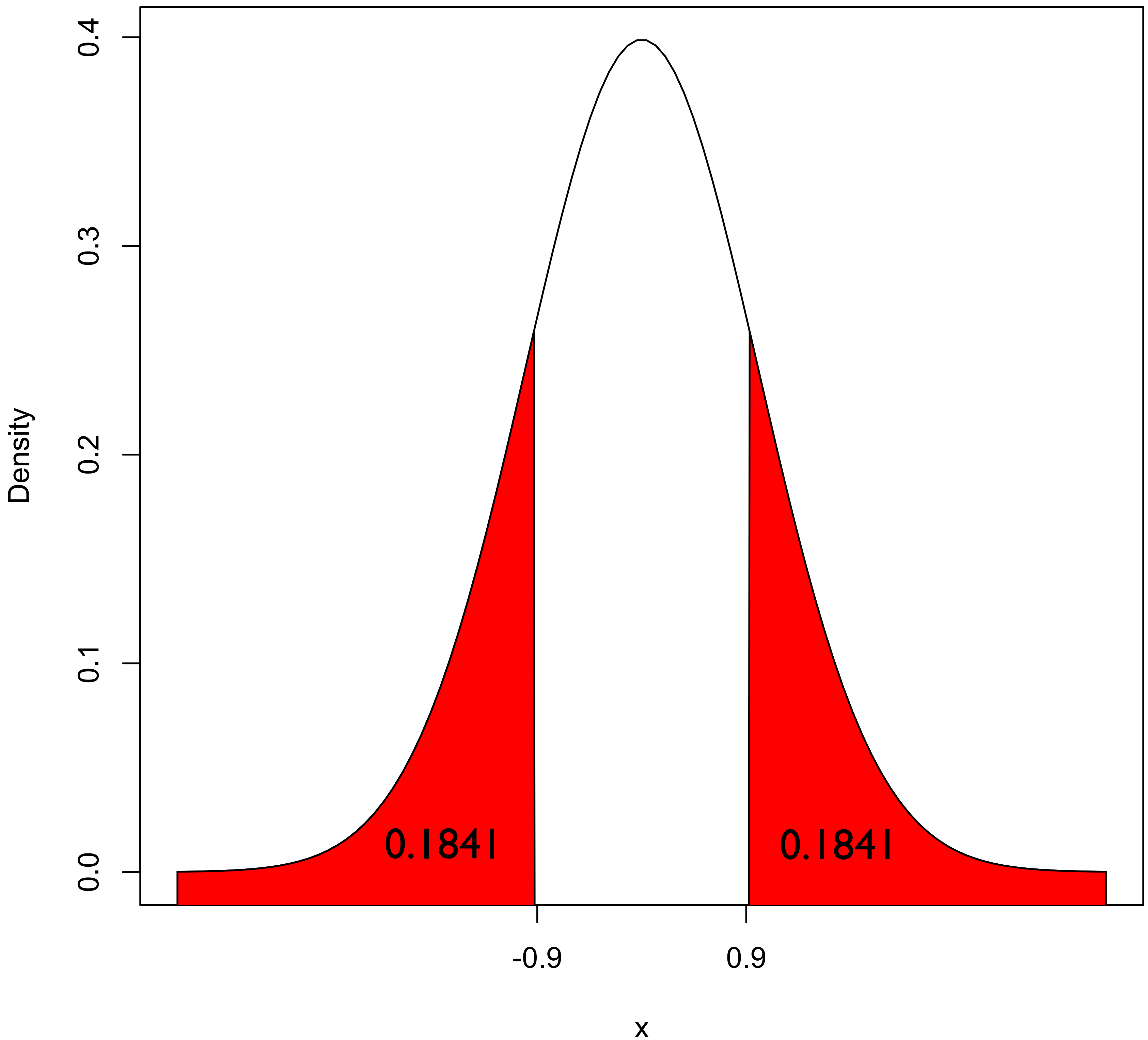
$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}}$$

4. P-value / critical region

- Step four can be done two different ways
- First need to decide on alpha (typically 0.05)
- Then proceed along one of two paths

P-value

- The first path is to calculate the p-value of the test statistic.
- What is the p-value? The probability of finding the sample statistic, or something more extreme, if the null were true.
- For two-tailed tests, double p-value from the table



Say you had a test statistic of 0.9 for a two-tailed test.

The red region represents the probability of finding a more extreme test statistic if the null were true

0.1841+0.1841 = 0.3682

P-value

- If your p-value is less than your alpha, then you reject the null
- A small p-value means its really unlikely to find your test statistic if the null were true
- More likely the null is false

P-value

- For example, say the null is that the population mean is 50
- Your sample mean is 100 with a standard deviation of 5 and n of 200
- Your test statistic is 141.42, with pvalue of nearly 0
- Much more likely that the true population mean is not 50

P-value

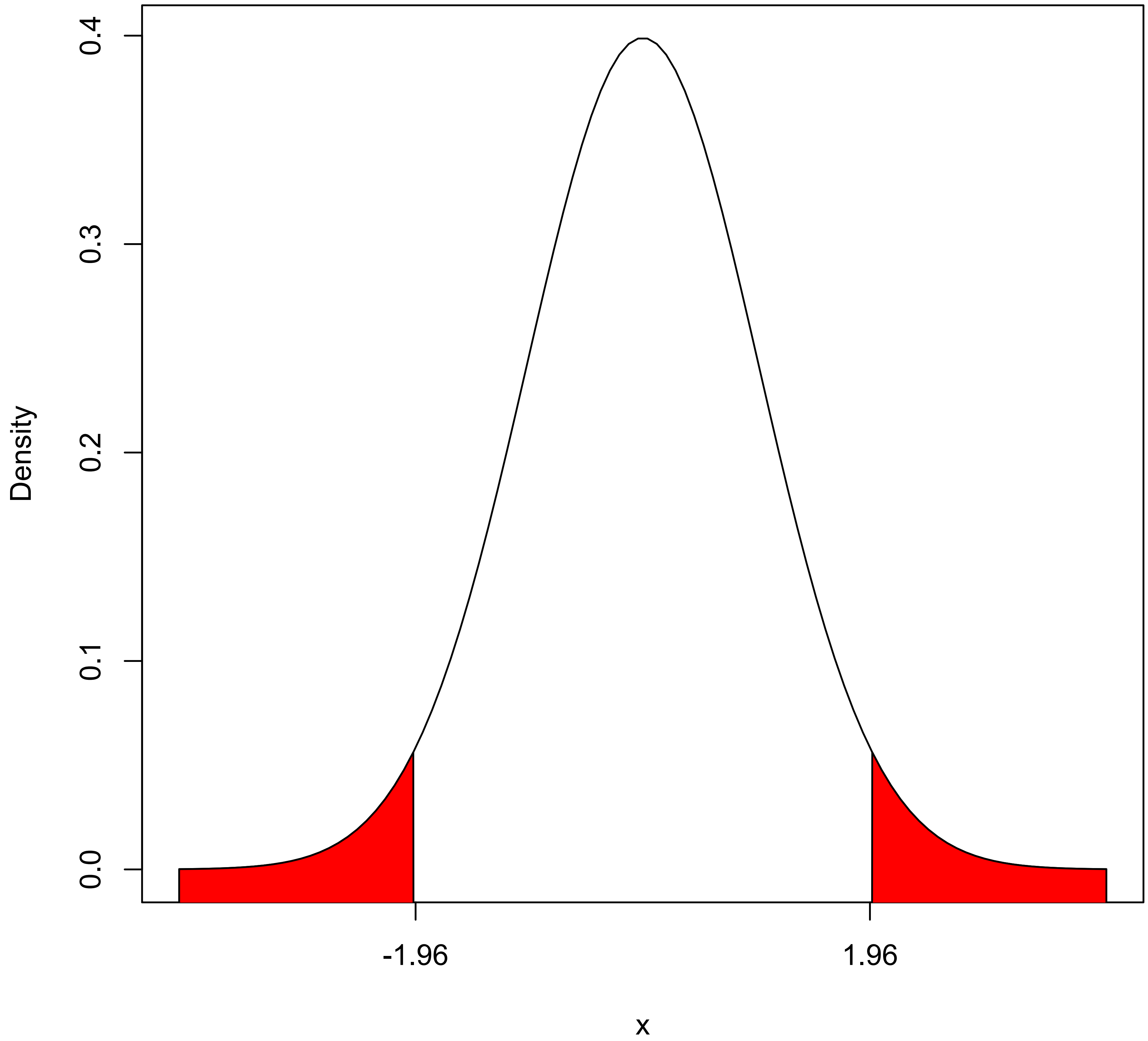
- We can think of the p-value as the chance of making a type 1 error given the current data

Critical Region

- The second pathway involves finding the t score that corresponds to your alpha
- So assuming a normal distribution, two-tailed test, and alpha of 0.05, the t score would be 1.96
- This is your critical value

Critical Region

- The critical region is the area under the distribution that is outside the critical value
- If our computed test statistic falls into this critical region, we reject the null hypothesis
- We can think of the critical region as the region of incredulity - a value in this region leads us to believe the null is not true.



0.4
0.3
0.2
0.1
0.0
Density
-1.96
1.96
x

Two Pathways

- The two pathways lead to the same conclusion - if our p-value is less than alpha, it follows that our test statistic is inside the critical region, and vice versa
- So we only need to use one method

5. Conclusion

- Once we've finished step 4, it's time to make a decision regarding our hypotheses

Decisions

- Only two possible decisions:
  - Reject the null in favor of the alternative
  - Fail to reject the null
- Notice: we never accept the null - many plausible values for the population mean

Conclusion

- Just like we did with confidence intervals, we need to interpret our decision
- Brief sentence or two describing the results of the test

Sample Problem

- Two-tailed test
- Your friend claims that the average UCI undergrad owns 30 non-textbook books. You don't believe him and to prove him wrong, you survey 100 undergrads. In your sample, you find a mean of 25 and standard deviation of 5. Was your friend right?

1. Assumptions

- Independent Random Sample
- Normally distributed sample distribution

2. Hypotheses

- Null: the mean number of books is 30

$$H_0 : \mu = 30$$

- Alternative: the mean number of books is not 30

$$H_A : \mu \neq 30$$

- Alpha = 0.05

3. Test Statistic

$$\bar{x} = 25$$
$$s = 5$$
$$\mu_0 = 30$$
$$n = 100$$

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}$$
$$= \frac{25 - 30}{\frac{5}{\sqrt{100}}}$$
$$= \frac{-5}{.5}$$
$$t = -10$$

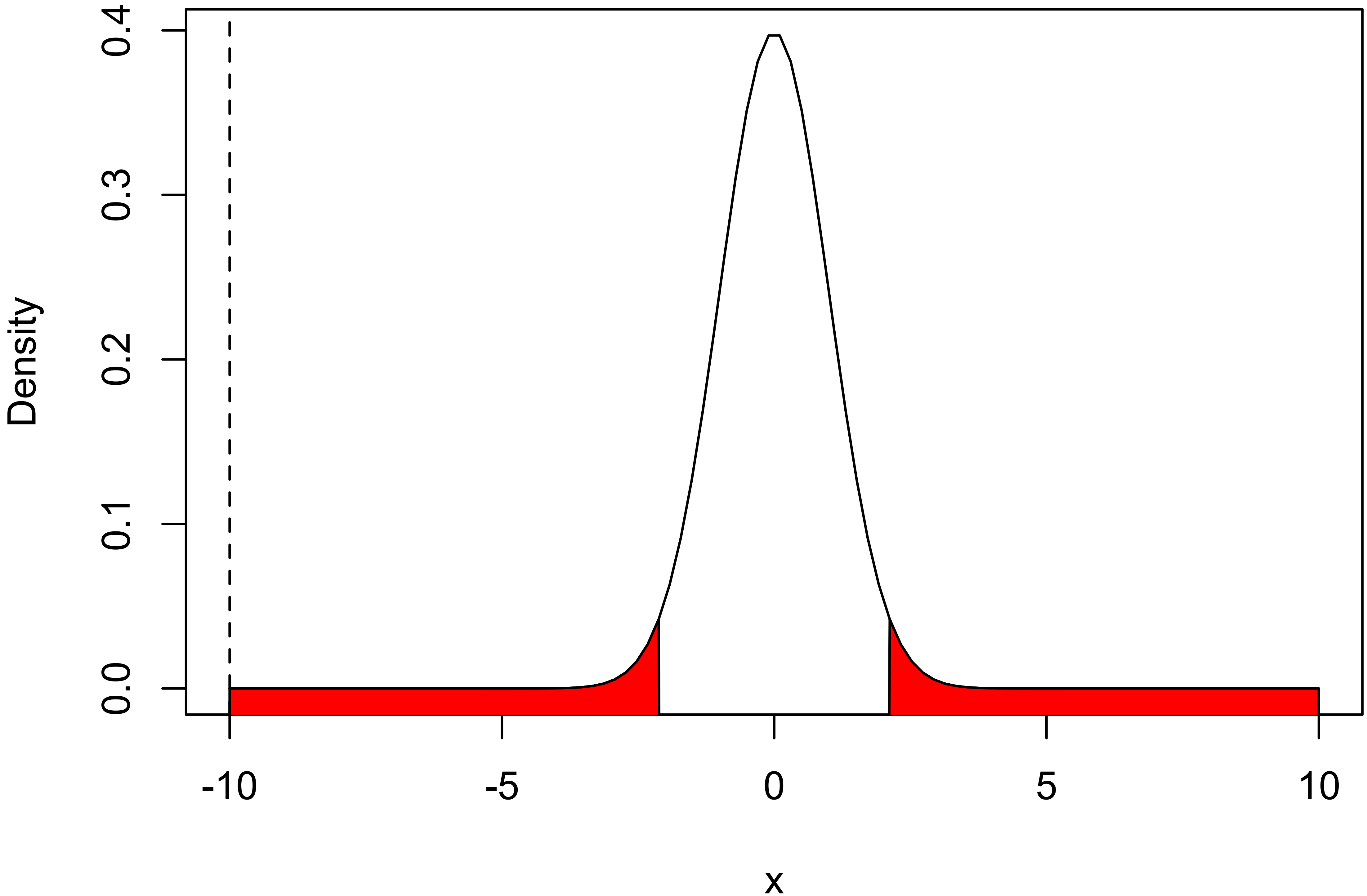
4. P-value

Second decimal place in z										
0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	z
									0.0000†	−3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	−3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	−3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	−3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	−3.5

According to the table, our p-value is essentially 0

4. Critical Region

- Critical value for alpha = 0.05 on a two-tailed test is ±1.96
- Our test statistic is well outside the critical value, deep in the critical region



Density
0.0
0.1
0.2
0.3
0.4
-10
-5
0
5
10
x

5. Conclusion

- Since our p-value is so low, and our test statistic is inside the critical region, we reject the null hypothesis in favor of the alternative hypothesis
- There is enough evidence to say that the mean number of books owned by UCI undergrads is not 30.

Practice!

- You have a random sample of 50 adults and the time they spent reading the newspaper. The sample mean reading time is 15 minutes with a standard deviation of 4 minutes.
- Using an alpha of 0.05, test whether the population mean is 14 minutes.

Assumptions

- Independent random sample
- Normally distributed sample distribution

Hypotheses

- Null: mean time spent reading the newspaper is 14 minutes

$$H_0 : \mu = 14$$

- Alternative: mean time spent reading the newspaper is not 18 minutes

$$H_A : \mu \neq 14$$

- Alpha: 0.05

Test Statistic

$$\mu_0 = 14$$
$$\bar{x} = 15$$
$$s = 4$$
$$n = 50$$

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}$$
$$= \frac{15 - 14}{\frac{4}{\sqrt{50}}}$$
$$= \frac{1}{.5657}$$
$$t = 1.768$$

P-value

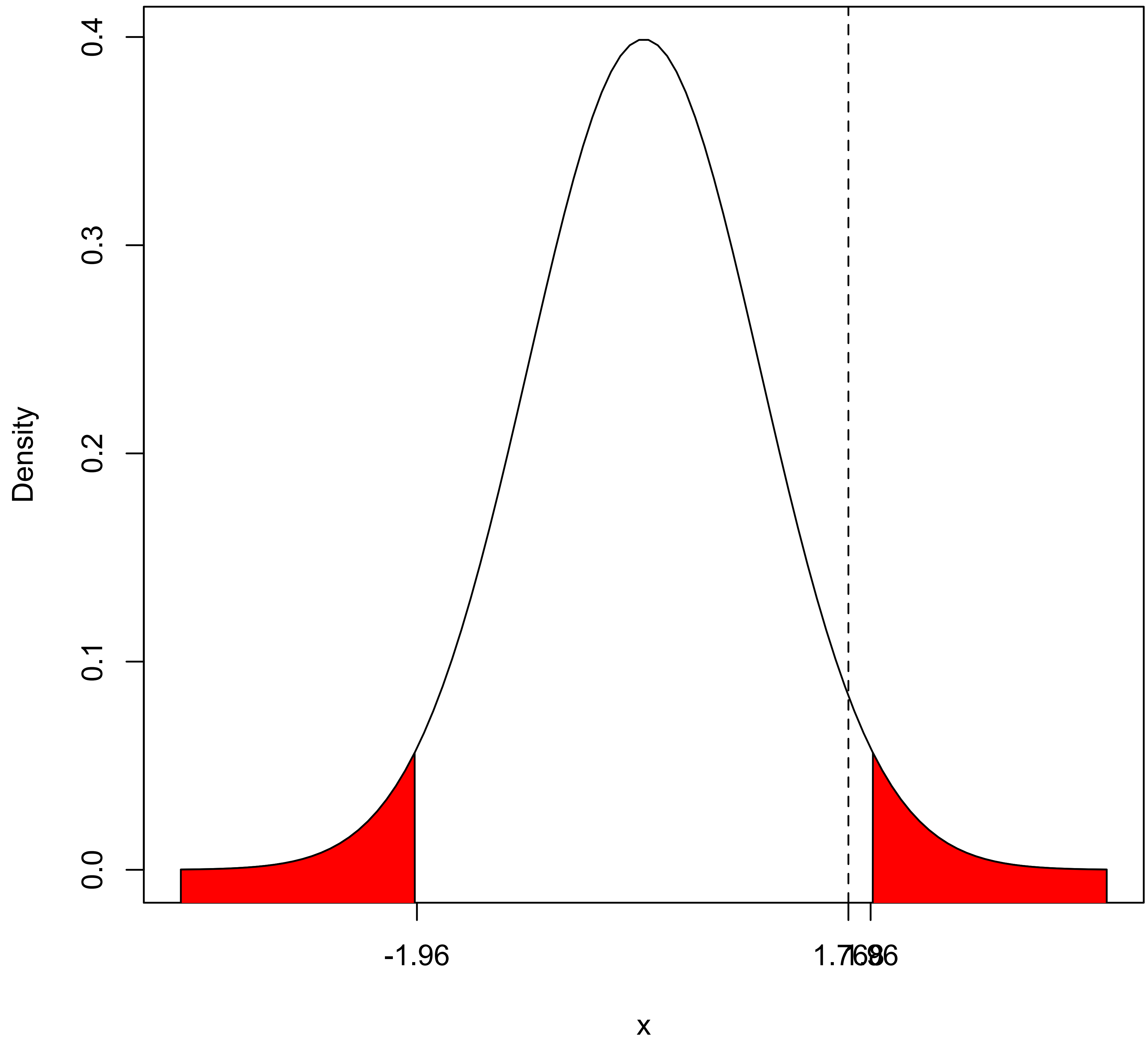
Second decimal place in z										
0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	z
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	−1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	−1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	−1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	−1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	−1.5

Two tailed test, so multiple by two

$$0.0384 * 2 = 0.0768$$

Critical Region

- For alpha of 0.05 in a two tailed test, critical value is 1.96
- Our test statistic, 1.768 is less than the critical value, it does not lie in the critical region


0.4
0.3
0.2
0.1
0.0
Density
-1.96
1.76
1.96
x

Conclusion

- With a test statistic of 1.768 and a p-value of 0.0768, we fail to reject the null hypothesis that the mean time spent reading the newspaper is 14 minutes.
- 15 minutes is a plausible sample mean if the population mean is 14 minutes.

One-tailed tests

- One tailed tests assume a direction - that the population mean is either less than or greater than some parameter.
- Remember: with one-tailed tests, you don't need to multiply the p-value from the table by two

Sample Problem

- Your friend is now trying to claim that most college students start drinking when they are 18 years old. You think it's higher. You sample 50 UCI students and ask when they started drinking. The average age was 18.8 with a standard deviation of 2.3. Who was correct? Use an alpha of 0.05

Assumptions

- Independent random sample
- Normal sampling distribution

Hypotheses

- Null: The average age when starting to drink is equal to 18

$$H_0 : \mu = 18$$

- Alternative: The average is is greater than 18

$$H_A : \mu > 18$$

- Alpha: 0.05

Test Statistic

$$\mu_0 = 18$$
$$\bar{x} = 18.8$$
$$s = 2.3$$
$$n = 50$$

$$
\begin{aligned}
t &= \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} \\
&= \frac{18.8 - 18}{\frac{2.3}{\sqrt{50}}} \\
&= \frac{0.8}{0.3253} \\
t &= 2.46
\end{aligned}
$$

P-value

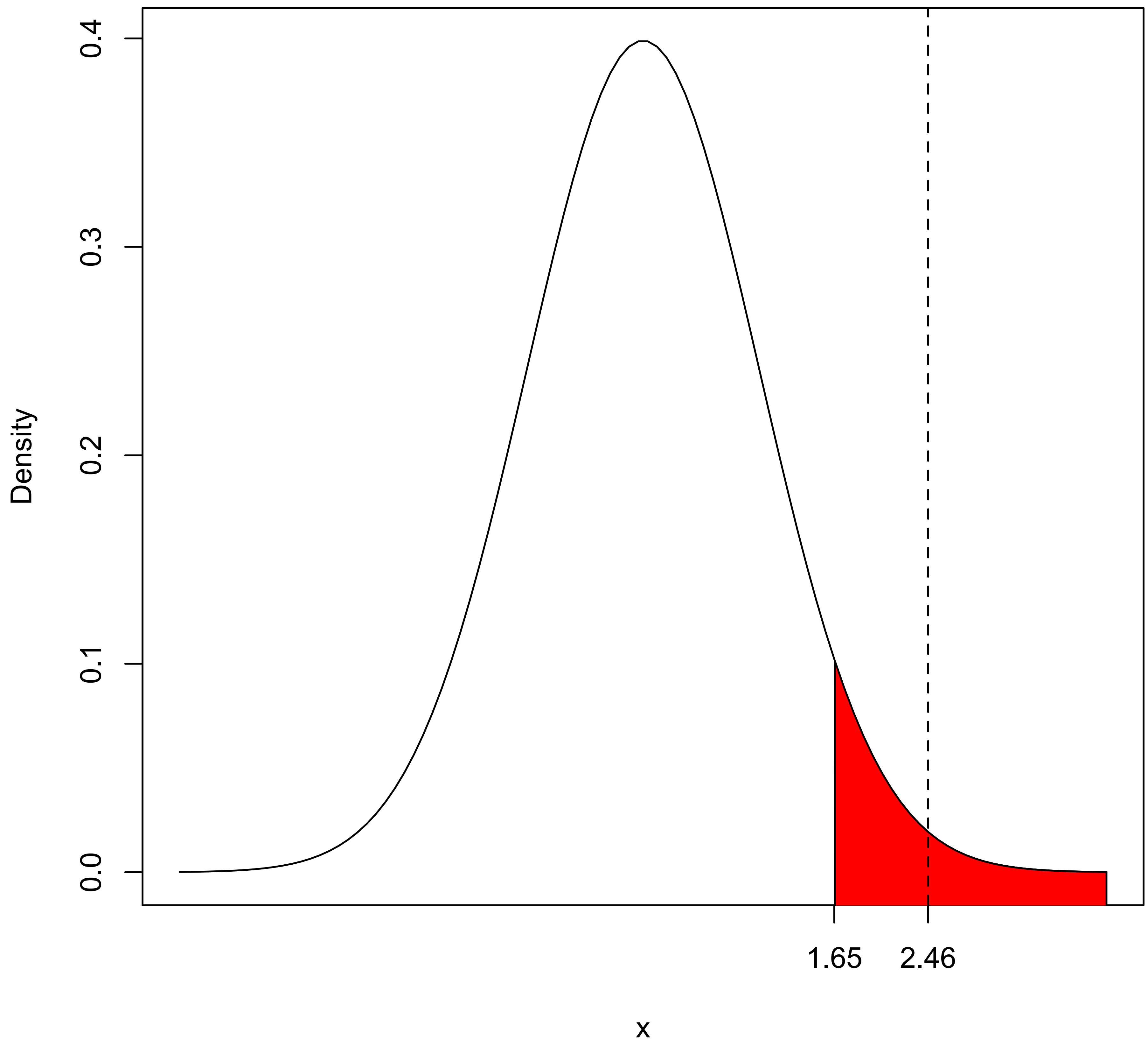
Second decimal place in z										
0.09	*0.08*	*0.07*	*0.06*	*0.05*	*0.04*	*0.03*	*0.02*	*0.01*	*0.00*	*z*
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	−2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	−2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	−2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	−2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	−2.0

This is a one-tailed test so we don't need to double it

$p = 0.0069$

Critical Region

- The critical value for alpha of 0.05 for a one tailed test is 1.65
- Our test statistic is 2.46, larger than the critical value
- This means our test statistic lies within the critical region



Density
0.0
0.1
0.2
0.3
0.4
1.65
2.46
x

Decision

- With a test statistic of 2.46 and a p-value of 0.0069, we reject the null hypothesis in favor of the alternative hypothesis.
- There is enough evidence to suggest that the mean age at which college students start drinking is greater than 18.

Practice!

- Your friend now claims that the average cost of the cars that undergrads drive is \$30,000. You think it is lower and to prove it you sample 15 undergrads and find an average car cost of \$25,000 with a standard deviation of \$15,000. Using an alpha of 0.05, test whether the average cost of an undergrad's car is less than \$50,000.

Assumptions

- Independent random sample
- Normal population distribution

Hypotheses

- Null: the average cost of an undergrad's car is $30,000

$$H_0 : \mu = 30,000$$

- Alternative: the average cost of an undergrad's car is less than $30,000

$$H_A : \mu < 30,000$$

Test Statistic

$$\mu_0 = 30,000$$
$$\bar{x} = 25,000$$
$$s = 15,000$$
$$n = 15$$
$$\alpha = 0.05$$

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}$$
$$= \frac{25,000 - 30,000}{\frac{15{,}000}{\sqrt{15}}}$$
$$= \frac{-5,000}{3872.98}$$
$$t = -1.29$$

P-Value

	80%	90%	95%	98%
			Right-Tail Probability	
df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$
1	3.078	6.314	12.706	31.821
2	1.886	2.920	4.303	6.965
3	1.638	2.353	3.182	4.541
4	1.533	2.132	2.776	3.747
5	1.476	2.015	2.571	3.365
6	1.440	1.943	2.447	3.143
7	1.415	1.895	2.365	2.998
8	1.397	1.860	2.306	2.896
9	1.383	1.833	2.262	2.821
10	1.372	1.812	2.228	2.764
11	1.363	1.796	2.201	2.718
12	1.356	1.782	2.179	2.681
13	1.350	1.771	2.160	2.650
14	1.345	1.761	2.145	2.624
15	1.341	1.753	2.131	2.602

So our t statistic corresponds to a p-value greater than 0.1, but we don't know exactly.

P-Value

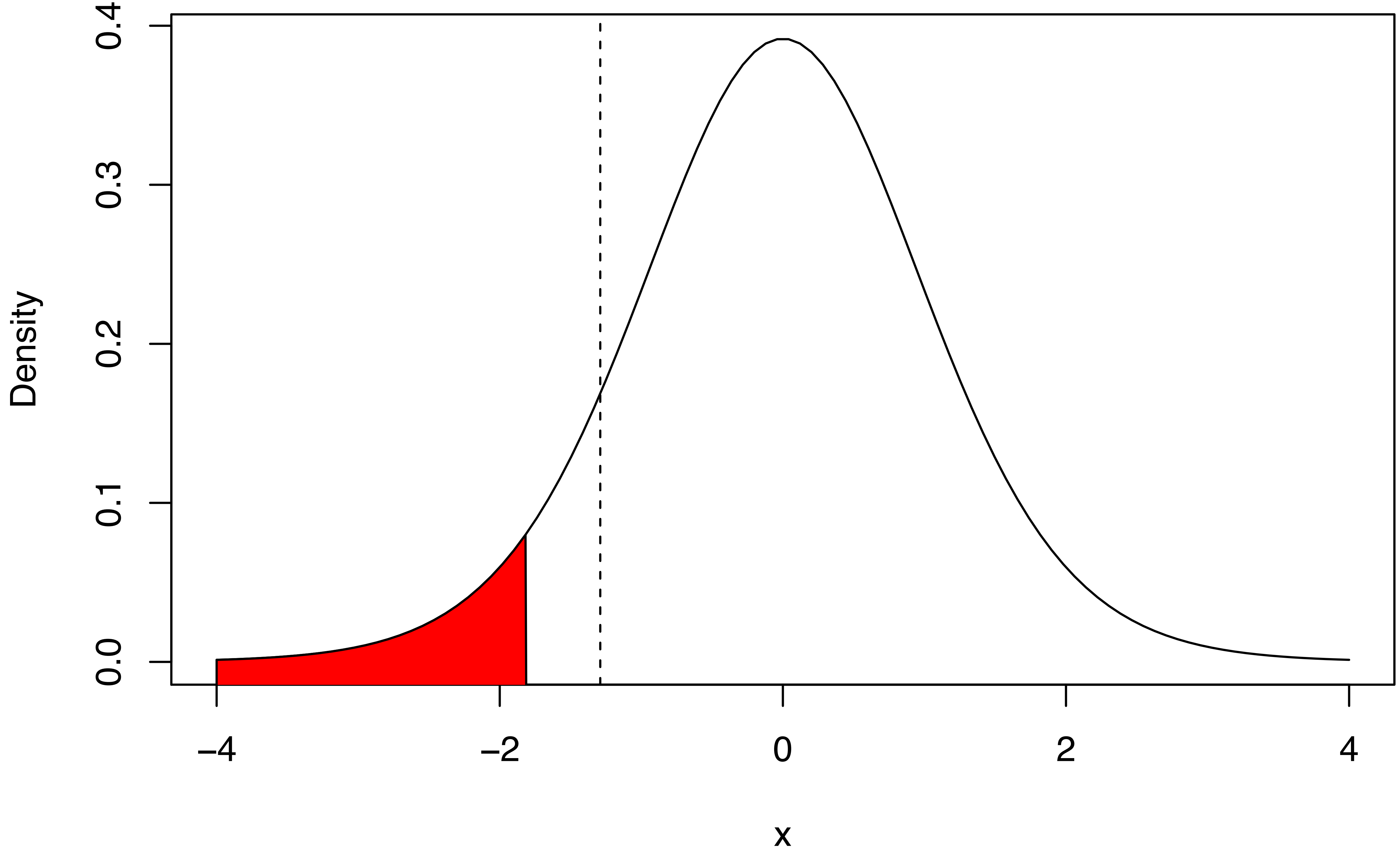
- This is an instance where the critical region is easier

Critical Region

	80%	90%	95%	98%
			Right-Tail Probability	
df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$
1	3.078	6.314	12.706	31.821
2	1.886	2.920	4.303	6.965
3	1.638	2.353	3.182	4.541
4	1.533	2.132	2.776	3.747
5	1.476	2.015	2.571	3.365
6	1.440	1.943	2.447	3.143
7	1.415	1.895	2.365	2.998
8	1.397	1.860	2.306	2.896
9	1.383	1.833	2.262	2.821
10	1.372	1.812	2.228	2.764
11	1.363	1.796	2.201	2.718
12	1.356	1.782	2.179	2.681
13	1.350	1.771	2.160	2.650
14	1.345	1.761	2.145	2.624
15	1.341	1.753	2.131	2.602

Critical Region

- So the critical value for a df = 14, alpha = 0.05, and a one tailed test is -1.761
- Our test statistic is -1.29, which is less than the critical value
- So our test statistic does not lie in the critical region


Density
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4
x

Decision

- With a test statistic of -1.29 and a critical value of -1.761, we fail to reject the null hypothesis.
- There is not enough evidence to conclude the average cost of an undergrad's car is less than $30,000

Hypothesis Tests for a Proportion

For Proportions

- Same procedure as the tests for a mean
- Slight variation in notation and such

Assumptions

- Independent random sample - trials are Bernoulli trials
- Sampling distribution is normal

Hypotheses

- Null Hypothesis

$$H_0 : \pi = \pi_0$$

- Alternative Hypothesis

$$H_A : \pi \neq \pi_0 \text{ or } \pi < \pi_0 \text{ or } \pi > \pi_0$$

Test Statistic

$$z = \frac{\hat{\pi} - \pi_0}{\sigma_{\hat{\pi}}}$$

$$\sigma_{\hat{\pi}} = \sqrt{\frac{\pi_0(1 - \pi_0)}{n}}$$

Notice we use z instead of t

That's because small proportion samples don't use the t-distribution

Notice we use the null value in the standard error calculation

Sample Problem

- You flip a coin 50 times and come up heads 30 times. Is the coin a trick coin?
- Use an alpha of 0.05

Assumptions

- Independent random sample
- Sample distribution is normal

Hypotheses

- Null: the probability of getting heads with this coin is 0.5

$$H_0 : \pi = 0.5$$

- Alternative: the probability of getting heads with this coin is not 0.5

$$H_A : \pi \neq 0.5$$

- Alpha: 0.05

Test statistic

$$\pi_0 = 0.5$$

$$n = 50$$

$$x = 30$$

$$\hat{\pi} = \frac{30}{50} = 0.6$$

$$z = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$

$$= \frac{0.6 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{50}}}$$

$$= \frac{0.1}{0.0707}$$

$$z = 1.414$$

P-value

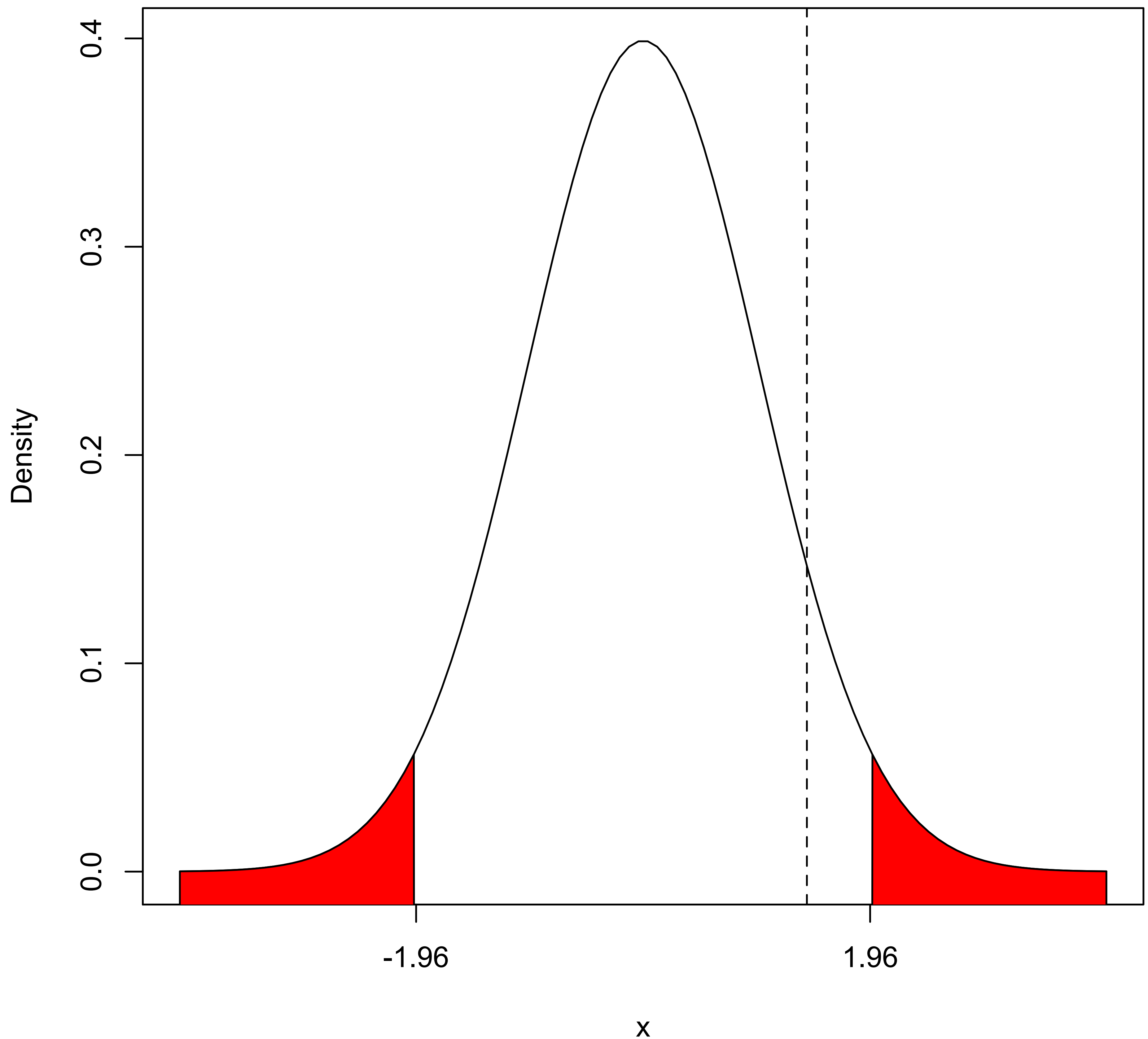
Second decimal place in z										
0.09	*0.08*	*0.07*	*0.06*	*0.05*	*0.04*	*0.03*	*0.02*	*0.01*	*0.00*	z
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	*−1.4*
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	*−1.3*
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	*−1.2*
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	*−1.1*
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	*−1.0*

This is a two-tailed test, so we need to double the p-value

$$0.0793 * 2 = 0.1586$$

Critical Region

- The critical value for an alpha of 0.05 in a two-tailed test is 1.96
- Our test statistic is 1.414, which is less than the critical value
- Therefore, our test statistic does not lie within the critical region



0.4
0.3
0.2
0.1
0.0
Density
-1.96
1.96
x

Decision

- With a test statistic of 1.414 and a p-value of 0.1586, we fail to reject the null hypothesis
- There is not enough evidence to claim that the coin is a trick coin - 30 heads in 50 flips is a plausible outcome of a fair coin.

Practice!

- Your friend claims that half of all UCI students have iPhones. You think that is far too large, and the true proportion is much smaller. You survey 100 students and find that 40 of them have iPhones. Is your friend right?
- Use an alpha of 0.05

Assumptions

- Independent Random Sample
- Sampling Distribution is Normal

Hypotheses

- Null: half of all UCI students have iPhones

$$H_0 : \pi = 0.5$$

- Alternative: less than half of all UCI students have iPhones

$$H_A : \pi < 0.5$$

- Alpha: 0.05

Test Statistic

$$\pi_0 = 0.5$$
$$n = 100$$
$$x = 40$$

$$\hat{\pi} = \frac{40}{100} = 0.4$$

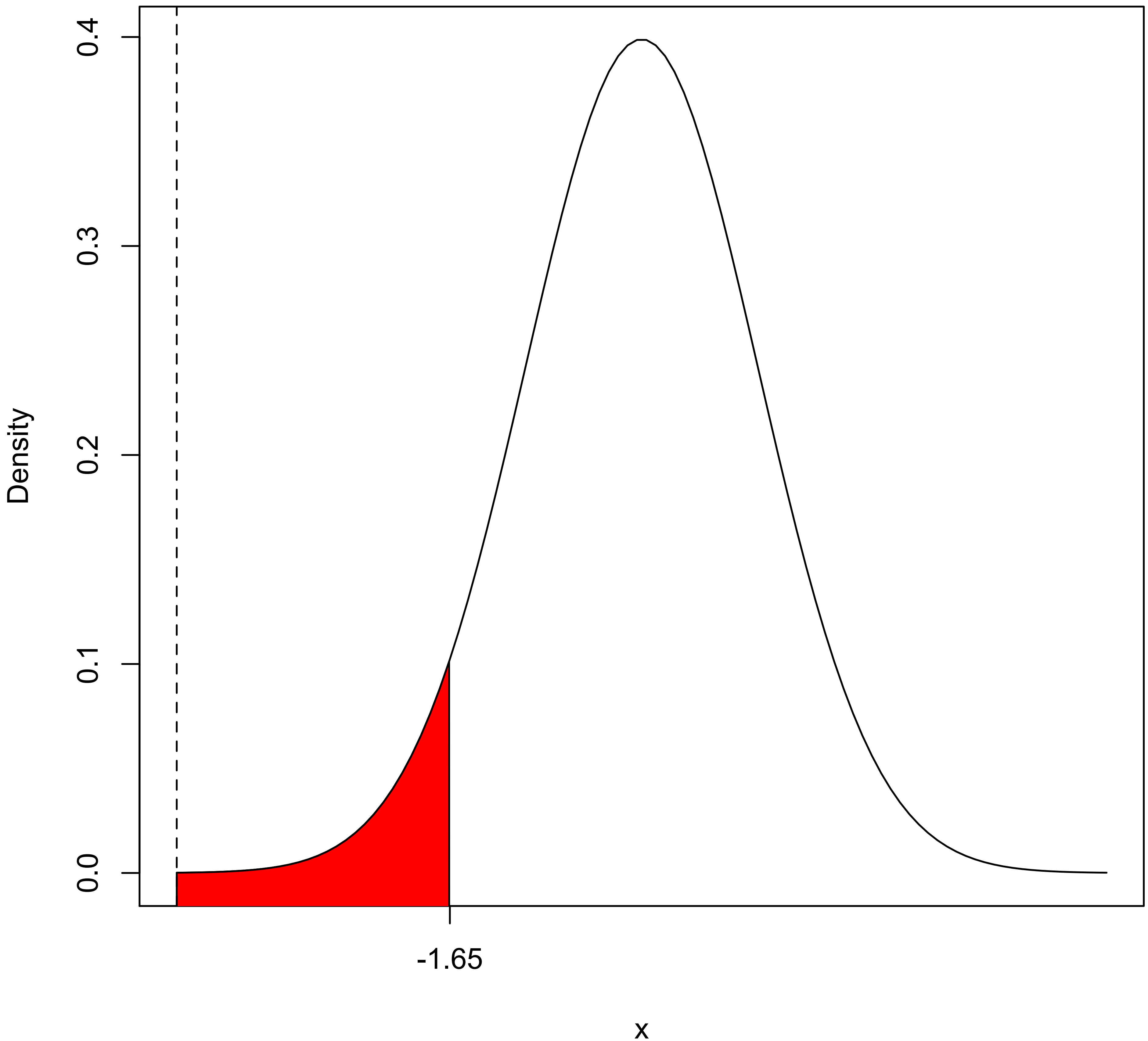
$$z = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$
$$= \frac{0.4 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{100}}}$$
$$= \frac{-0.1}{0.025}$$
$$z = -4$$

P-value

- The p-value is essentially 0

Critical Region

- The critical value for an alpha of 0.05 for a one tailed test is -1.65
- Our test statistic is -4, which is much lower than the critical value
- Therefore our test statistic is within the critical region


0.4
0.3
0.2
0.1
0.0
Density
-1.65
x

Decision

- With a test statistic of -4 and a p-value of approximately 0, we reject the null hypothesis in favor of the alternative.
- There is enough evidence to say that the proportion of UCI students with an iPhone is less than 0.5

One more time!

- You're friend is huffy that you've proven him wrong, and claims only 2 out of every 5 people can tolerate you. You counter that you are very likable, thank you very much, and you'll prove it. After asking 50 of your acquaintances, you find fully 25 people willing to admit they enjoy your company. At at alpha of 0.05, do more than 2 out of every 5 people like you?

Assumptions

- Independent random sample
- Sampling distribution is normal

Hypotheses

- Null: only 2 out of 5 people can tolerate you

$$H_0 : \pi = 0.4$$

- Alternative: more than 2 out of 5 people enjoy your company

$$H_A : \pi > 0.4$$

Test Statistic

$$\pi_0 = 0.4$$
$$x = 25$$
$$n = 50$$
$$\alpha = 0.05$$

$$\hat{\pi} = \frac{25}{50} = 0.5$$

$$Z = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$
$$= \frac{0.5 - 0.4}{\sqrt{\frac{0.4(1-0.4)}{50}}}$$
$$= \frac{0.1}{\sqrt{\frac{0.24}{50}}}$$
$$= \frac{0.1}{0.0693}$$
$$Z = 1.443$$

P-Value

Second decimal place in z										
0.09	*0.08*	*0.07*	*0.06*	*0.05*	*0.04*	*0.03*	*0.02*	*0.01*	*0.00*	*z*
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	*−1.4*
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	*−1.3*
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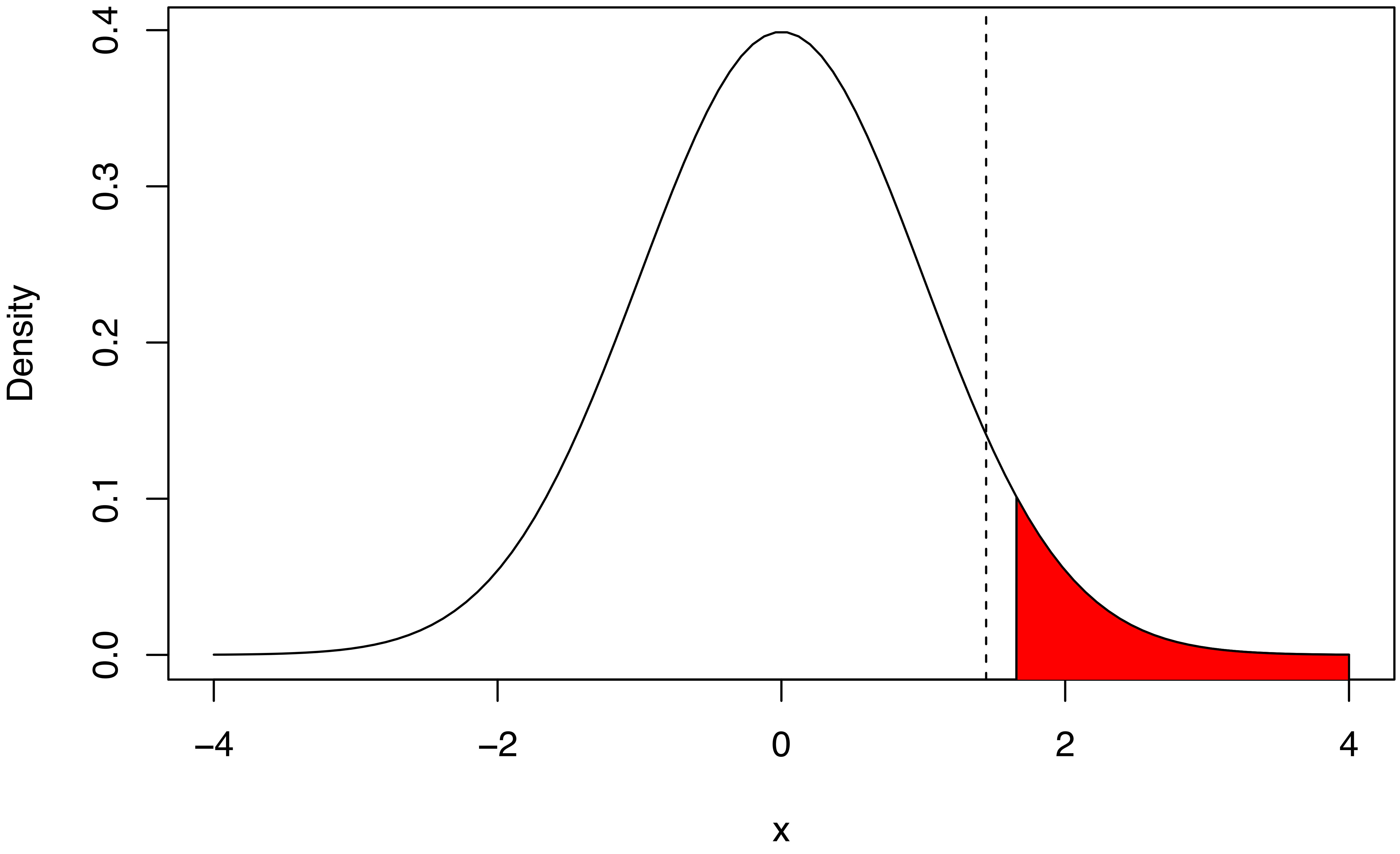
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	*−1.1*
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	*−1.0*

The p-value associated with our test statistic is 0.0749. Since this is a one tailed test, we do NOT need to double it.

Critical Region

- For a one tailed test with an alpha of 0.05, the critical value is 1.65
- Our test statistic is 1.443, which is less than the critical value
- Therefore, our test statistic does not lie in the critical region



0.0
0.1
0.2
0.3
0.4
Density
−4
−2
0
2
4
x

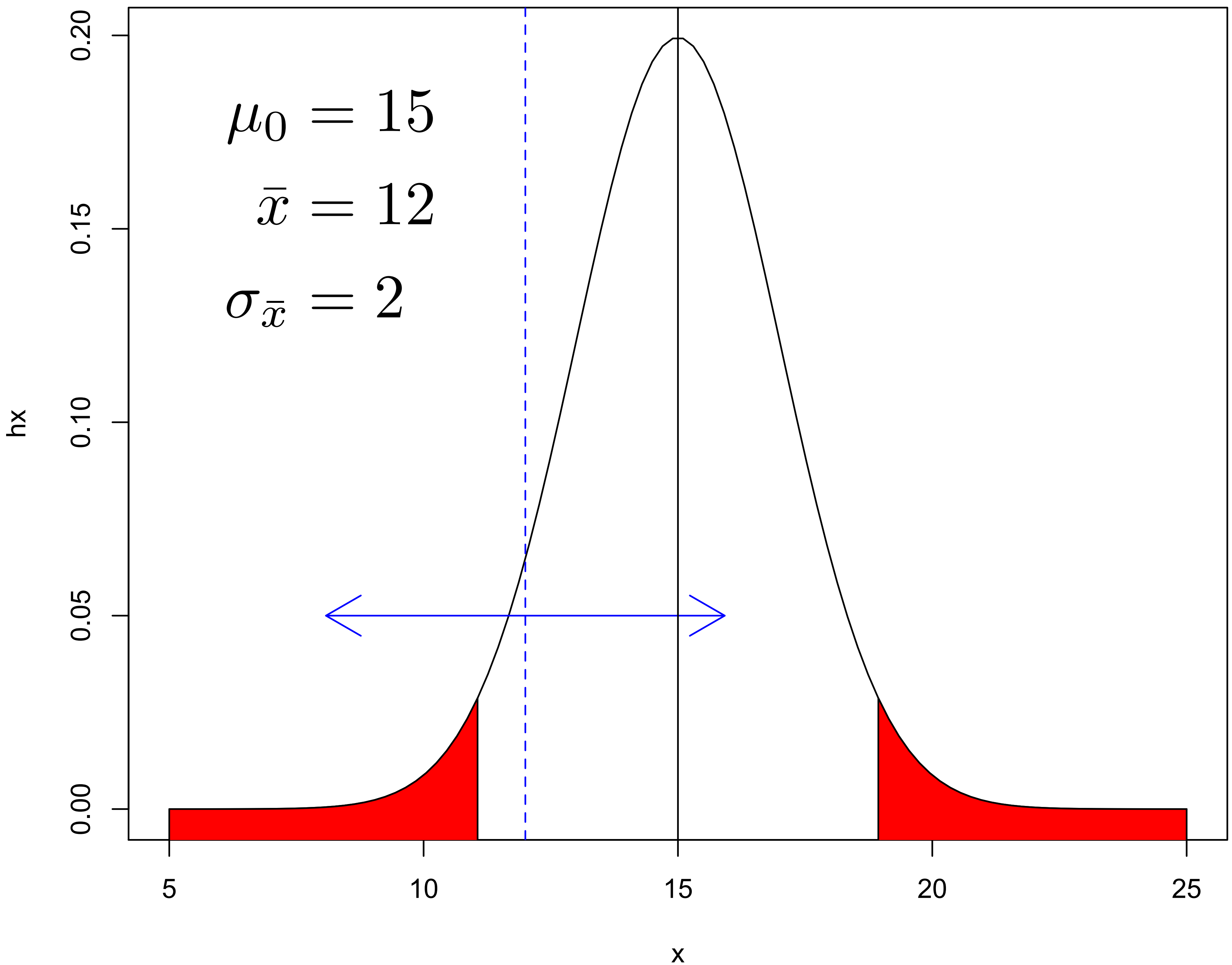
Decision

- With a p-value of 0.0749 and a test statistic of 1.443, we fail to reject the null hypothesis
- There is not enough evidence to claim that more than 2 out of every 5 people can tolerate you. You feel sad, until you remember you are awesome at stats and 40% of your acquaintances think you are cool.

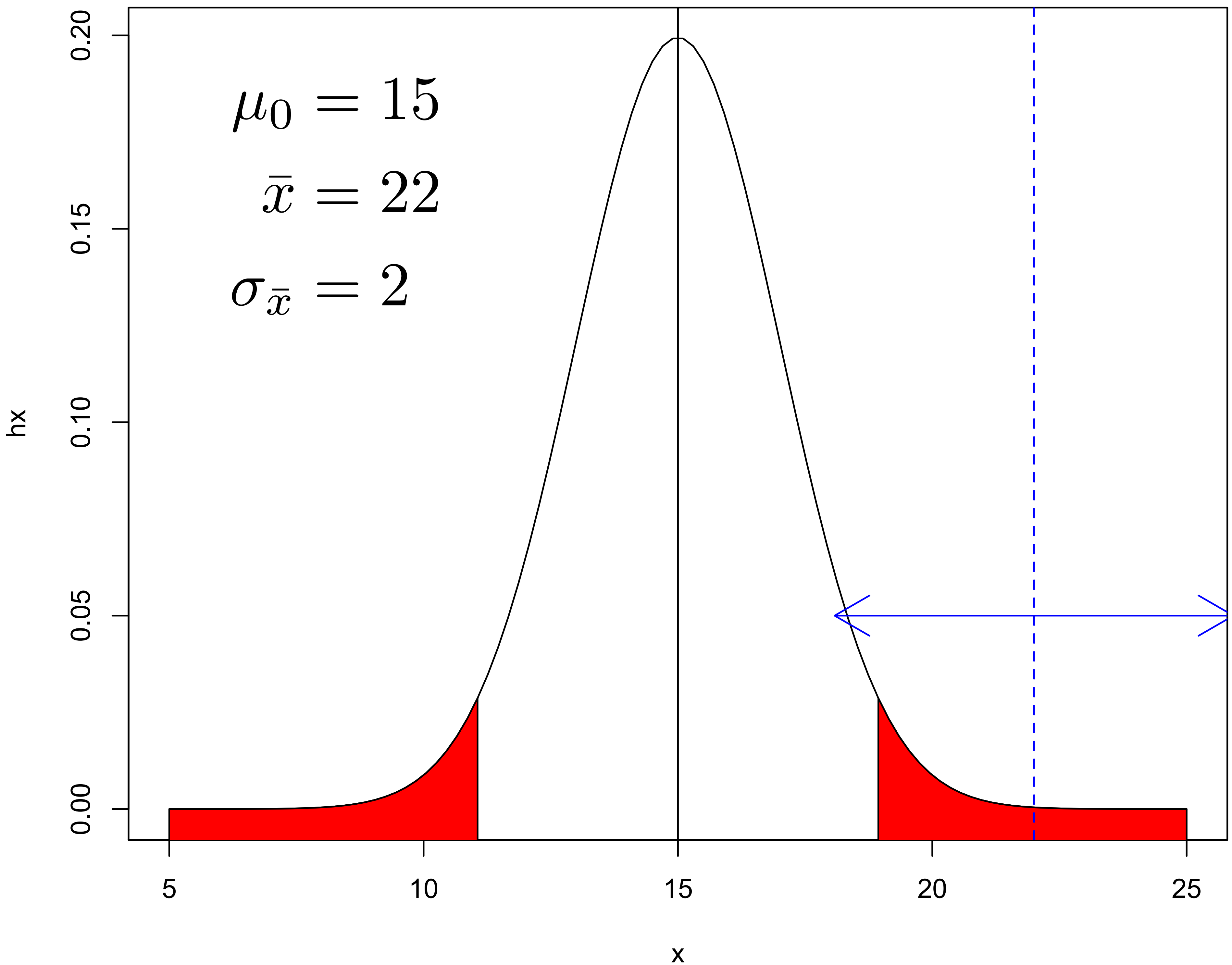
Confidence Intervals and Hypothesis Testing

- If you've been paying close attention, you may have noticed we use very similar procedures for confidence intervals and hypothesis testing
- In fact, 95% confidence intervals replicate two-tailed tests with alpha = 0.05

- If the null value is within the 95% confidence interval, you will fail to reject the null in a two-tailed test at alpha = 0.05
- If the null value is outside the 95% confidence interval, you will reject the null



$\mu_0 = 15$
$\bar{x} = 12$
$\sigma_{\bar{x}} = 2$
hx
0.00
0.05
0.10
0.15
0.20
5
10
15
20
25
x



$\mu_0 = 15$
$\bar{x} = 22$
$\sigma_{\bar{x}} = 2$
hx
0.00
0.05
0.10
0.15
0.20
5
10
15
20
25
x

Sample Problem

- By now, your friend finds your obsession with empirical testing of drunken debates disturbing and wonders how far he could push it...

- Your friend cautiously claims that the men's restroom at the local truck stop sees, on average, five sexual encounters between strangers per day. You shrug and spend the next month at the truck stop monitoring the men's restroom collecting data. At the end of the month, you witnessed, on average, 6.2 anonymous sexual encounters with a standard deviation of 3.1. Was your friend right, assuming an alpha of 0.05?

Assumptions

- Independent random sample
- Population is normally distributed

Hypotheses

- Null: the mean daily number of anonymous sexual encounters in the men's restroom at the local truck stop is 5

$$H_0 : \mu = 5$$

- Alternative: the mean number of encounters is not 5

$$H_A : \mu \neq 5$$

Test Statistic

$$\mu_0 = 5$$
$$\bar{x} = 6.2$$
$$s = 3.1$$
$$n = 30$$
$$\alpha = 0.05$$

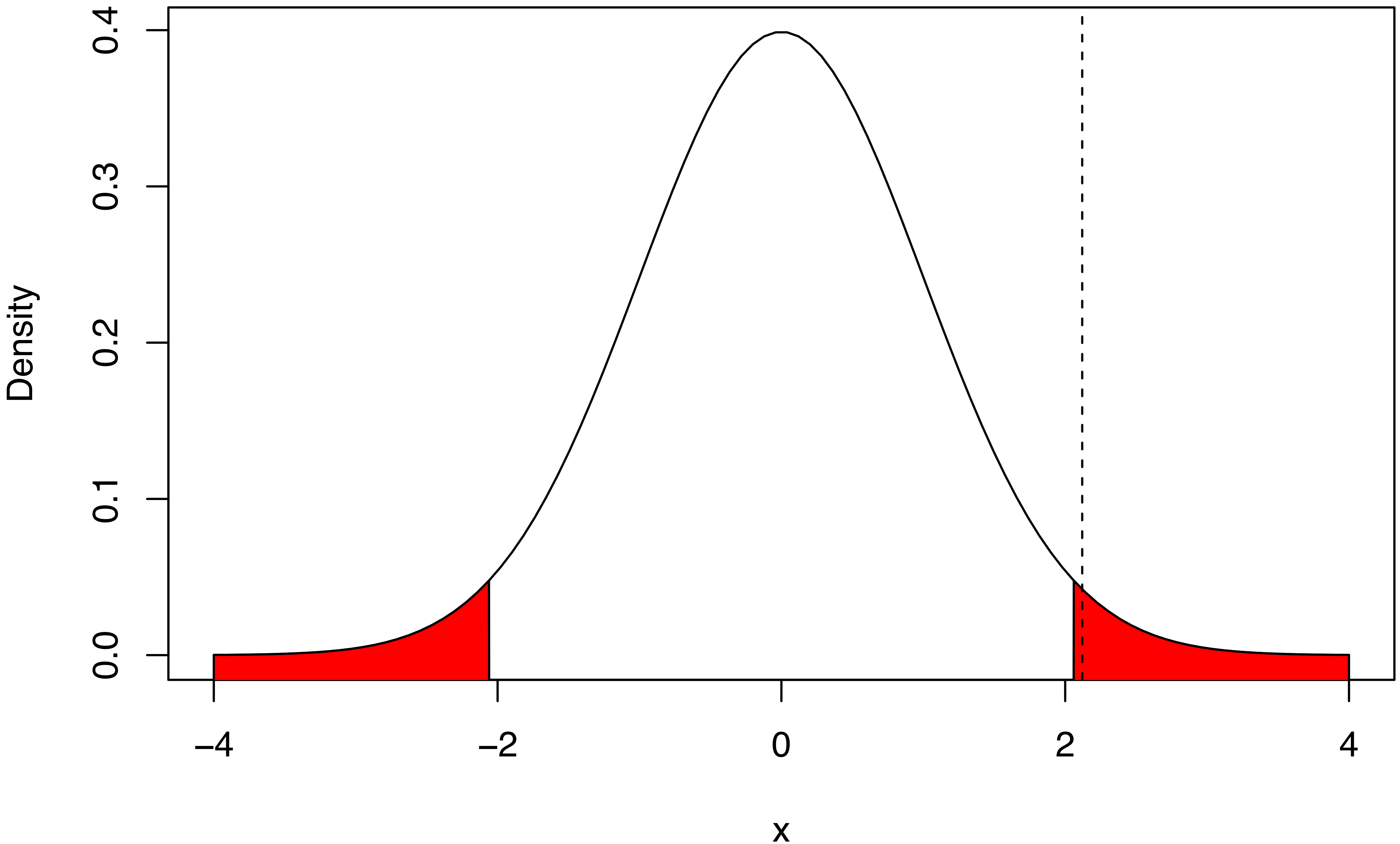
$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$
$$= \frac{6.2 - 5}{\frac{3.1}{\sqrt{30}}}$$
$$= \frac{1.2}{0.566}$$
$$t = 2.12$$

Critical Region

	Confidence Level					
	80%	90%	95%	98%	99%	99.8%
	Right-Tail Probability					
df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
40	1.303	1.684	2.021	2.423	2.704	3.307
50	1.299	1.676	2.009	2.403	2.678	3.261
60	1.296	1.671	2.000	2.390	2.660	3.232
80	1.292	1.664	1.990	2.374	2.639	3.195

Critical Region

- For a two tailed test with alpha of 0.05 and df = 29, the critical value is 2.045
- Our test statistic is 2.12, which is more extreme than the critical value
- Thus, our test statistic lies within the critical region


0.0
0.1
0.2
0.3
0.4
Density
−4
−2
0
2
4
x

Decision

- With a test statistic of 2.12 compared to a critical value of 2.045, we reject the null in favor of the alternative.
- We have enough evidence to claim that the average number of daily sexual encounters between strangers in the men's room at the local truck stop is not 5.

Confidence Interval

- What are plausible values for the average daily number of anonymous sexual encounters in the men’s restroom?

$$\bar{x} = 6.2$$
$$s = 3.1$$
$$n = 30$$
$$t = 2.045$$

$$\bar{x} \pm t\left(\frac{s}{\sqrt{n}}\right)$$

$$6.2 \pm 2.045\left(\frac{3.1}{\sqrt{30}}\right)$$
$$6.2 \pm 2.045(0.566)$$
$$6.2 \pm 1.116$$

We are 95% confident that the average number of daily sexual encounters between strangers in the men's restroom at the local truck stop is between 5.084 and 7.316 encounters per day