Statistics Refresher Course

Day 3

Two Samples

Two Samples

- Up until now, we've been working with single samples
- But what it is much more interesting to compare samples
  - Do men and women do the same amount of housework?
  - Do people who exercise live longer than people who don't?

Two Samples

- Samples can be dependent, or independent
- Dependent samples means that whether someone is in one sample depends on who is in the other sample
- Comparing husbands and wives, for example

Independent Samples

- Independent samples - who is in one sample has nothing to do with who is in a second sample

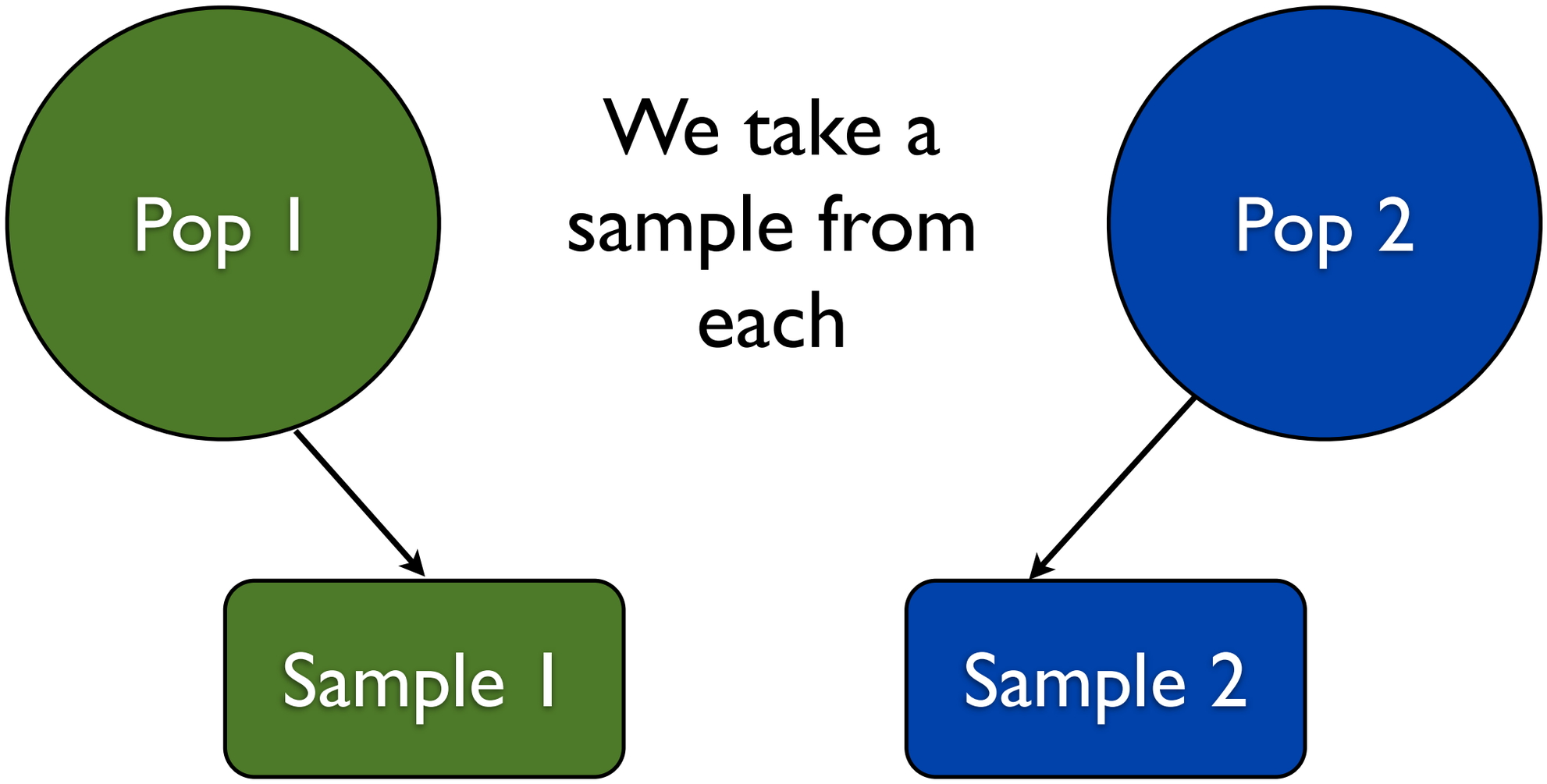
What can we do with two samples?

- Calculate the mean for each
- Calculate the difference in means
- Test whether means are statistically different

Notation

	Sample 1	Sample 2
Population mean	μ_1	μ_2
Sample Mean	$\bar{X}_1$	$\bar{X}_2$
Sample Size	N_1	N_2

Consider two populations



$\bar{X}_1 = 23.5$ Calculate sample means $\bar{X}_2 = 25.2$

$$\bar{X}_1 - \bar{X}_2 = 23.5 - 25.2 = -1.7$$

What are plausible values for the population mean difference?

Confidence Intervals

- We can construct a confidence interval for the difference between two populations
- But to do so, we need to know what the sampling distribution is for the difference between two means

Sampling Distribution

- If we take multiple samples from each of the two populations and compute their means, we'll eventually create a distribution of possible differences
- This is exactly like the sampling distribution for a single mean

Assumptions

- Large Sample
- Independent random sample
- Interval level dependent variable
- Nominal level independent variable with two categories
- Normally distributed sampling distribution

Dependent Variable

- Dependent variable is the variable you think is influenced by other factors
- Value *depends* on values of independent variable(s)

Independent Variable

- Variable you think influences the outcome of the dependent variable
- In this case, defines what distinguishes the two samples

Sampling Distribution for Difference

Mean: $\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$

Standard Error: $\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\sigma_1^2 + \sigma_2^2}$

$$= \sqrt{\frac{s_1^2}{N_1 - 1} + \frac{s_2^2}{N_2 - 1}}$$

Confidence Interval

$$(\bar{X}_1 - \bar{X}_2) \pm Z\sigma_{\bar{X}_1 - \bar{X}_2}$$

$$(\bar{X}_1 - \bar{X}_2) \pm Z\sqrt{\frac{s_1^2}{N_1 - 1} + \frac{s_2^2}{N_2 - 1}}$$

You are interested in who are better students: anthropologists or sociologists. You collect some data and calculate the following data

	Anthro	Sociol
mean GPA	3.2	3.6
st dev	0.8	1.2
n	150	125

Construct a 95% confidence interval for the population mean difference between sociologists' and anthropologists' GPAs.

$$\bar{X}_1 = 3.6$$
$$\bar{X}_2 = 3.2$$
$$s_1 = 1.2$$
$$s_2 = 0.8$$
$$n_1 = 125$$
$$n_2 = 150$$

$$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{s_1^2}{N_1 - 1} + \frac{s_2^2}{N_2 - 1}}$$
$$= \sqrt{\frac{1.2^2}{125 - 1} + \frac{0.8^2}{150 - 1}}$$
$$= \sqrt{\frac{1.44}{124} + \frac{0.64}{149}}$$
$$= \sqrt{0.0116 + 0.00430}$$
$$= 0.126$$

$$(\bar{X}_1 - \bar{X}_2) \pm Z\sigma_{\bar{X}_1 - \bar{X}_2}$$
$$(3.6 - 3.2) \pm 1.96(0.126)$$
$$0.4 \pm 0.247$$

We are 95% confident that the population mean difference between sociologists' and anthropologists' GPAs is between 0.153 and 0.647

- Someone suggests that people who drink coffee make more money than people who don't. You collect some data and calculate the following statistics:

	Coffee Drinkers	Coffee Abstainers
mean income	34,200	33,100
st dev	7000	6500
n	200	200

Construct a 95% confidence interval for the population mean difference in income between coffee drinkers and abstainers

$$\bar{X}_1 = 34,200$$
$$\bar{X}_2 = 33,100$$
$$s_1 = 3000$$
$$s_2 = 2300$$
$$n_1 = 200$$
$$n_2 = 200$$

$$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{s_1^2}{N_1 - 1} + \frac{s_2^2}{N_2 - 1}}$$

$$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{7000^2}{200 - 1} + \frac{6500^2}{200 - 1}}$$

$$= \sqrt{\frac{49000000}{199} + \frac{42250000}{199}}$$

$$= \sqrt{246231.16 + 212311.56}$$

$$= 677.16$$

$$(\bar{X}_1 - \bar{X}_2) \pm Z\sigma_{\bar{X}_1 - \bar{X}_2}$$

$$(34200 - 33100) \pm 1.96(677.16)$$

$$1100 \pm 1327.23$$

We can be 95% confident that the population mean difference in income between coffee drinkers and coffee abstainers is between $-227.23 and $2427.23

Confidence Intervals

- Notice that in the previous example, 0 was a plausible value for the population difference in means.
- This implies that if we do a hypothesis test, we won't find statistical support for the claim that coffee drinkers make more than abstainers

Hypothesis Test

- We do a hypothesis test for the difference between means just as we did with only one sample
- This time, though, the test statistic we calculate is Z, and is representative of the difference in means

Assumptions

- Large sample (small samples use different equations)
- Independent random samples
- Normally distributed sampling distribution
- Interval level dependent variable, nominal level independent variable

Null Hypothesis

- When testing difference, the null is always no difference:

$$H_0 : \mu_1 = \mu_2$$

$$H_0 : \mu_1 - \mu_2 = 0$$

Alternative Hypothesis

- Two-tailed: $H_A : \mu_1 \neq \mu_2 \quad \mu_1 - \mu_2 \neq 0$

- One-tailed: $H_A : \mu_1 > \mu_2 \quad \mu_1 - \mu_2 > 0$

  $H_A : \mu_1 < \mu_2 \quad \mu_1 - \mu_2 < 0$

Test Statistic

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sigma_{\bar{X}_1 - \bar{X}_2}}$$

	Coffee Drinkers	Coffee Abstainers
mean income	34,200	33,100
st dev	7000	6500
n	200	200

Using the same data as before, test for whether coffee drinkers make more than coffee abstainers, using an alpha of 0.05

Assumptions

- Large sample
- Independent random samples
- Normally distributed sampling distribution
- Interval level dependent variable, nominal level independent variable

Hypotheses

- Null: coffee drinkers and abstainers make the same income

$$H_0 : \mu_1 = \mu_2$$

- Alternative: coffee drinkers make more than coffee abstainers

$$H_A : \mu_1 > \mu_2$$

Test Statistic

$$\bar{X}_1 = 34200$$
$$\bar{X}_2 = 33100$$
$$\sigma_{\bar{X}_1 - \bar{X}_2} = 677.16$$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sigma_{\bar{X}_1 - \bar{X}_2}}$$

$$Z = \frac{(34200 - 33100) - 0}{677.16}$$
$$= \frac{1100}{677.16}$$
$$Z = 1.624$$

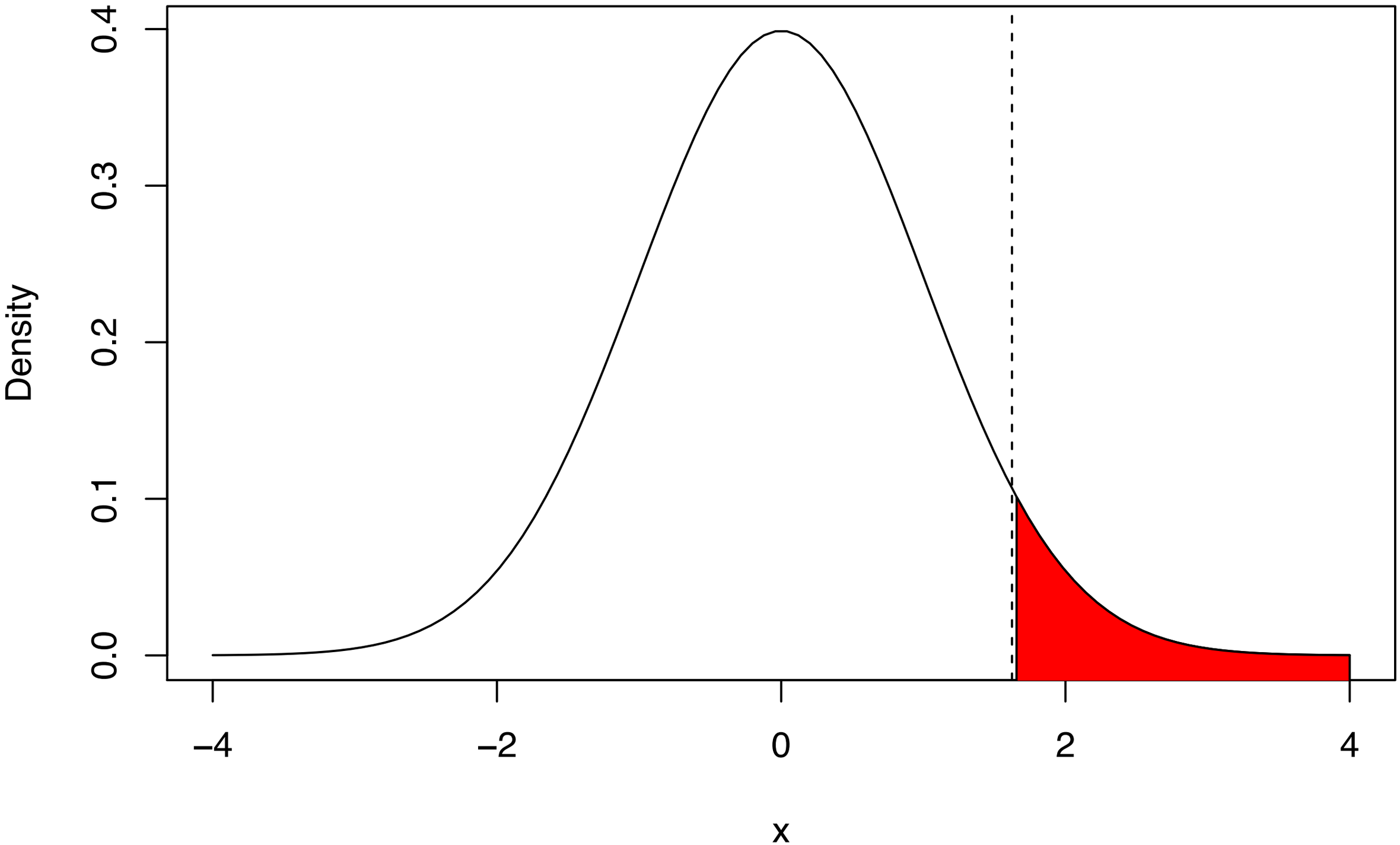
P-Value

Second decimal place in z										
0.09	*0.08*	*0.07*	*0.06*	*0.05*	*0.04*	*0.03*	*0.02*	*0.01*	*0.00*	z
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	*−1.9*
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	*−1.8*
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	*−1.7*
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	*−1.6*
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	*−1.5*

So our p-value is 0.0526. Since this is a one tailed test, we do not need to multiply by two. Our p-value is greater than our alpha (0.05)

Critical Region

- For a one tailed test, with a large sample and an alpha of 0.05, our critical value is 1.65
- Our test statistic is 1.624, which is smaller than the critical value
- Our test statistic does not fall in the critical region


Density
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4
x

Conclusion

- With a p-value of 0.0526 and a test statistic of 1.624, we fail to reject the null hypothesis.
- There is not enough evidence to claim that coffee drinkers make more money than coffee abstainers

Two Proportions

Two Proportions

- When both variables are nominal
  - Gender and opinion on the death penalty

Notation

	Sample 1	Sample 2
Population Proportion	π_1	π_2
Sample Proportion	$\hat{\pi}_1$	$\hat{\pi}_2$
Sample Size	N_1	N_2

Sampling Distribution

- Normally distributed as long as you have large samples
- Often people say at least 5 observations in each category

Sampling Distribution

- Mean $\mu_{\hat{\pi}_1 - \hat{\pi}_2} = (\hat{\pi}_1 - \hat{\pi}_2)$

- Standard Error

$$\sigma_{\hat{\pi}_1 - \hat{\pi}_2} = \sqrt{\frac{\pi(1-\pi)}{N_1} + \frac{\pi(1-\pi)}{N_2}}$$

Standard Error

$$\sigma_{\hat{\pi}_1 - \hat{\pi}_2} = \sqrt{\frac{\pi(1-\pi)}{N_1} + \frac{\pi(1-\pi)}{N_2}}$$

- But what if we don't know π?
- Pooled estimate:

$$\hat{\pi} = \frac{N_1\hat{\pi}_1 + N_2\hat{\pi}_2}{N_1 + N_2}$$

Standard Error

- Remember: we assume the null hypothesis is true until proven otherwise
- The null hypothesis in this case is:
  - $\pi_1 = \pi_2$
- For that to be true, given the observed successes and cases, then the computed, pooled population estimate is the best estimate for the population proportion

- The following data come from the 2010 GSS, is gay sex wrong?:

	Wrong	Not Wrong	Total
Male	349	210	559
Female	355	309	664

Test the hypothesis that women are more supportive of gays than men, using an alpha of 0.05

Assumptions

- Independent random samples
- Normally distributed sampling distribution
- Nominal dependent and independent variables

Hypotheses

- Null: $H_0 : \mu_M = \mu_F$

- Alternative: $H_A : \mu_M < \mu_F$

Test Statistic

- Need to calculate some proportions

$$\hat{\pi}_M = \frac{210}{559} = 0.376$$

$$\hat{\pi} = \frac{N_M \hat{\pi}_M + N_F \hat{\pi}_F}{N_M + N_F}$$

$$\hat{\pi} = \frac{559 * 0.376 + 664 * 0.465}{559 + 664}$$

$$= \frac{519}{1223}$$

$$= 0.424$$

$$\sigma_{\hat{\pi}_1 - \hat{\pi}_2} = \sqrt{\frac{\pi(1-\pi)}{N_1} + \frac{\pi(1-\pi)}{N_2}}$$

$$\sigma_{\hat{\pi}_1 - \hat{\pi}_2} = \sqrt{\frac{0.424(1-0.424)}{559} + \frac{0.424(1-0.424)}{664}}$$

$$= \sqrt{\frac{0.2442}{559} + \frac{0.2442}{664}}$$

$$= 0.02837$$

$$Z = \frac{(\hat{\pi}_F - \hat{\pi}_M) - 0}{\sigma_{\hat{\pi}_F - \hat{\pi}_M}}$$

$$= \frac{(0.465 - 0.376) - 0}{0.02837}$$

$$Z = 3.137$$

P-Value

- The p-value of 3.137 is essentially 0, which is less than our alpha of 0.05

Critical Region

- For a one tailed test with alpha of 0.05, the critical value is 1.65
- Our test statistic is 3.137, which is greater than the critical value
- Therefore, our test statistic lies within the critical region

Conclusion

- With a p-value of essentially 0 and a test statistic of 3.137, we reject the null hypothesis in favor of the alternative
- Women are more likely to be supportive of homosexuality than men.

Binomial Distributions

Binomial Distribution

- Family of distributions based on Bernoulli trials
- B(n,p) where n is the number of trials and p is the probability of success

Binomial Distribution

- Ranges from 0 to n

- Mean $\mu = n * p$

- Standard Deviation $\sigma = \sqrt{np(1-p)}$

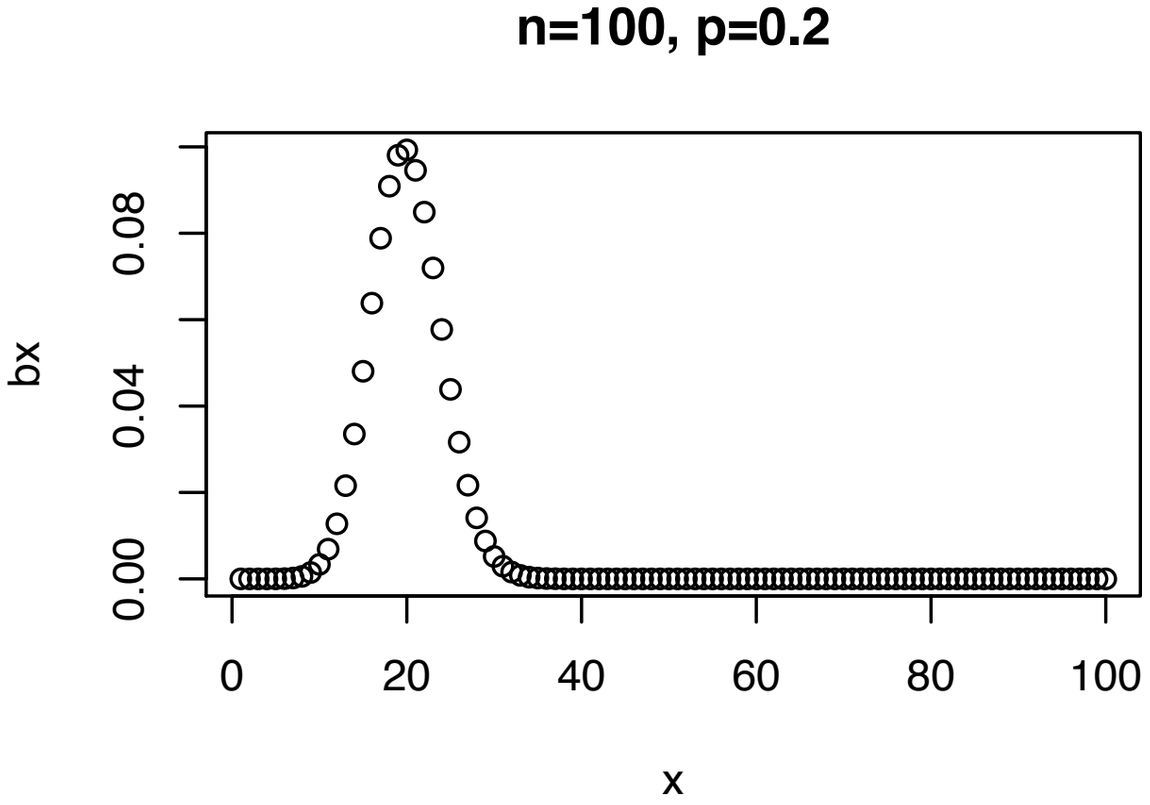
Binomial Distribution

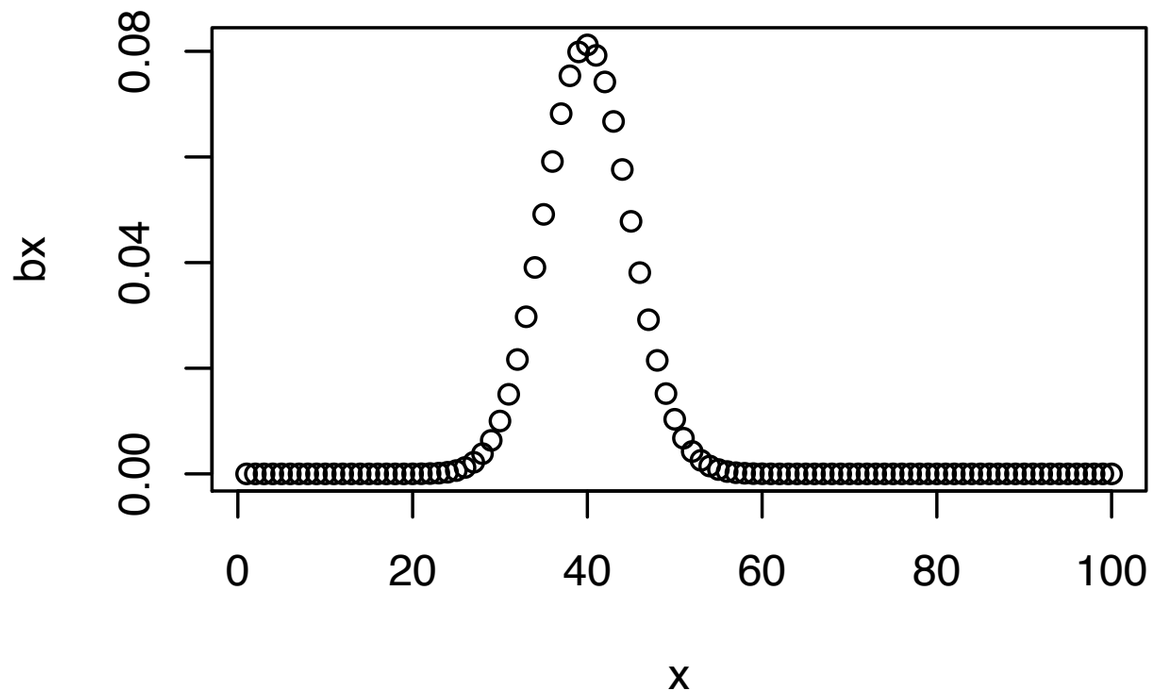
- Probability mass function (height)

$$f(k:n,p) = Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

- Cumulative distribution function (p-value)

$$F(k:n,p) = Pr(X \leq k) = \sum_{i=0}^{\lfloor x \rfloor} \binom{n}{i} p^i (1-p)^{n-i}$$


n=100, p=0.2
bx
0.00
0.04
0.08
0
20
40
60
80
100
x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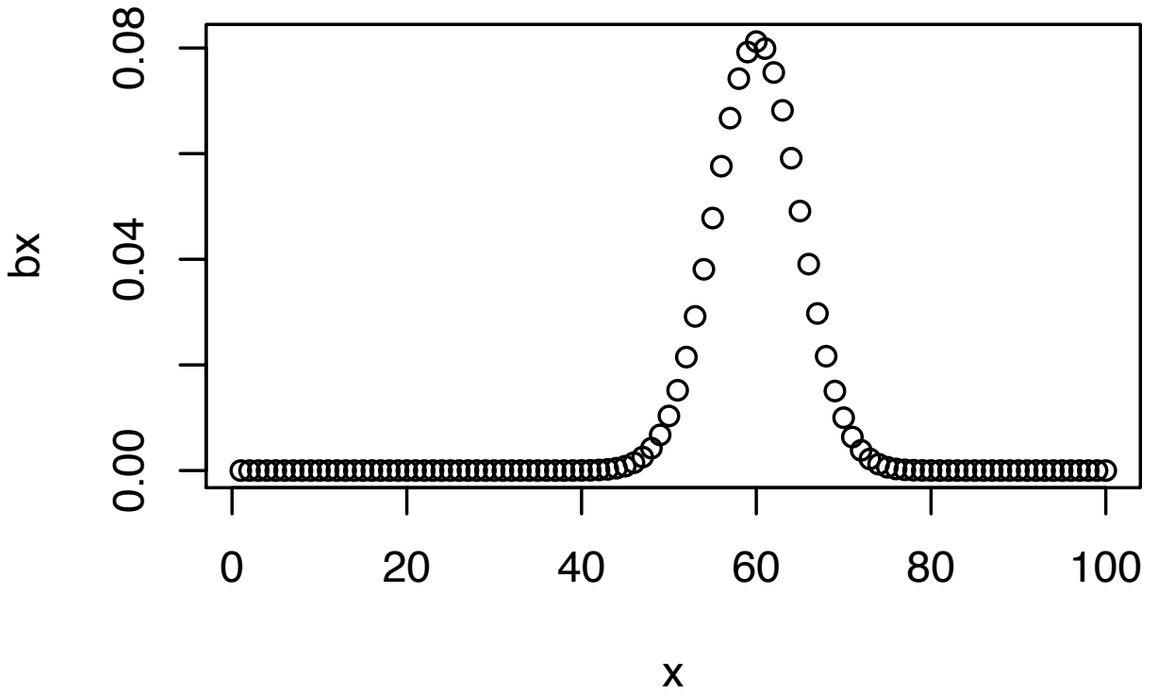
60
80
100
x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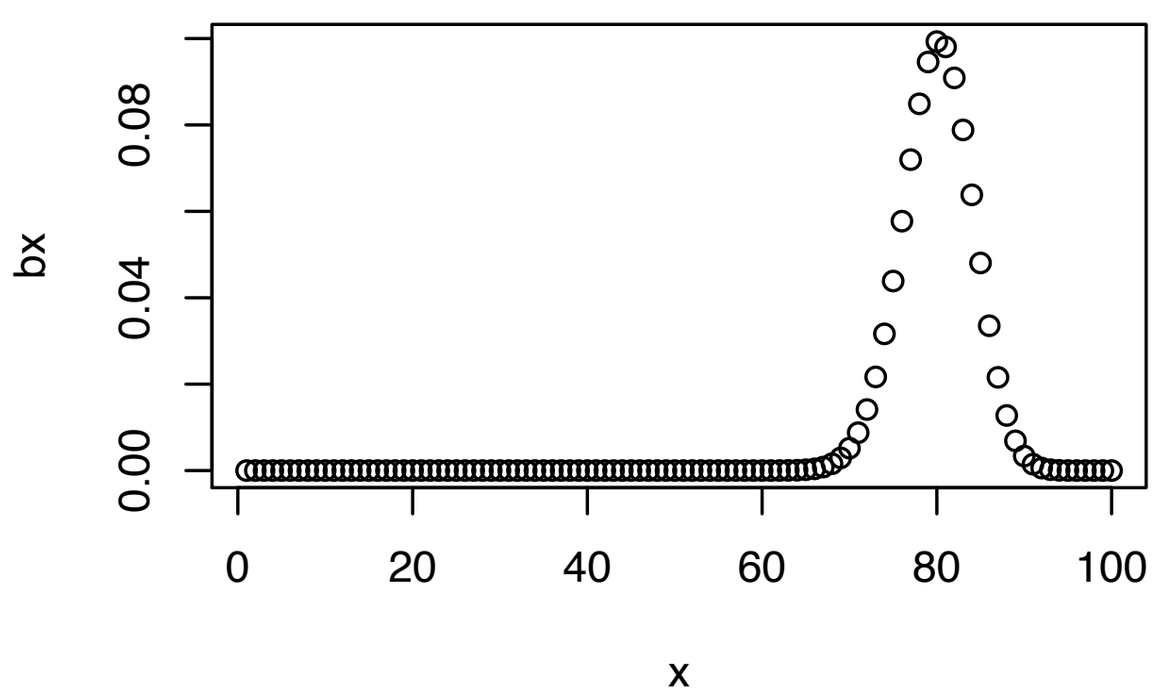
n=100, p=0.8
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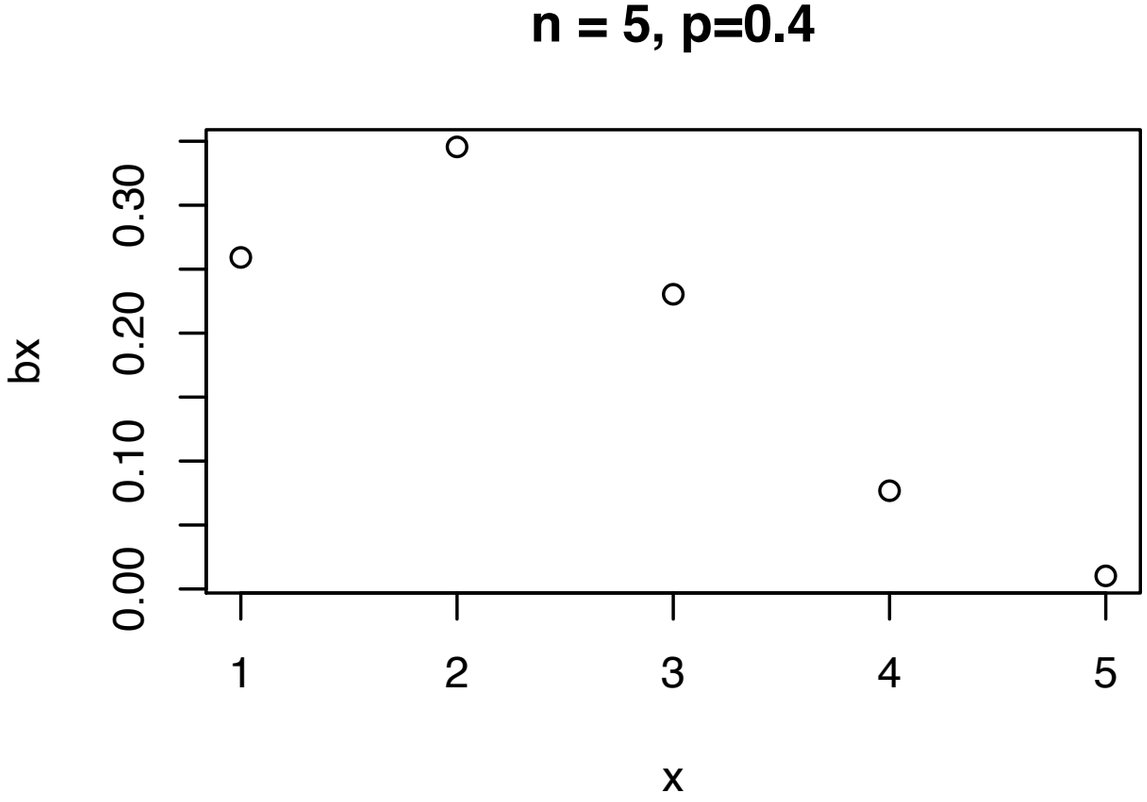
0.04
0.08
0
20
40
60
80
100
x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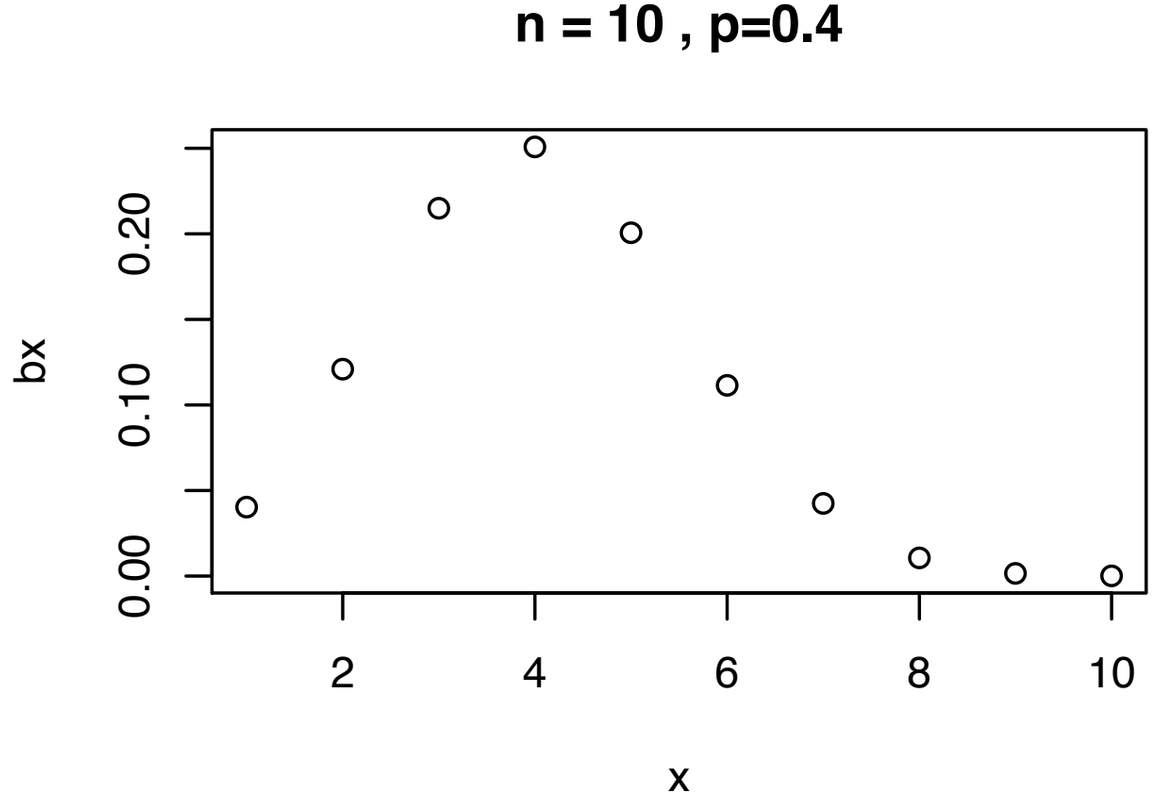
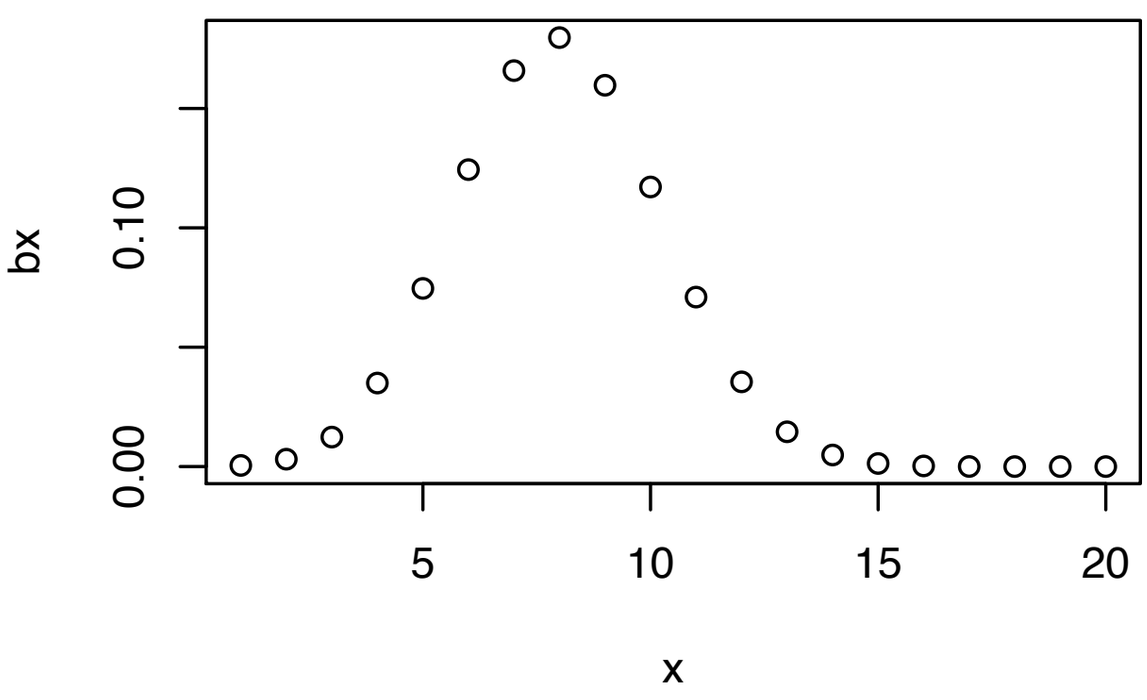
bx
0.00
0.10
0.20
0.30
1
2
3
4
5
x


n = 10 , p=0.4
bx
0.00
0.10
0.20
2
4
6
8
10
x


n = 20 , p=0.4
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0.00
0.10
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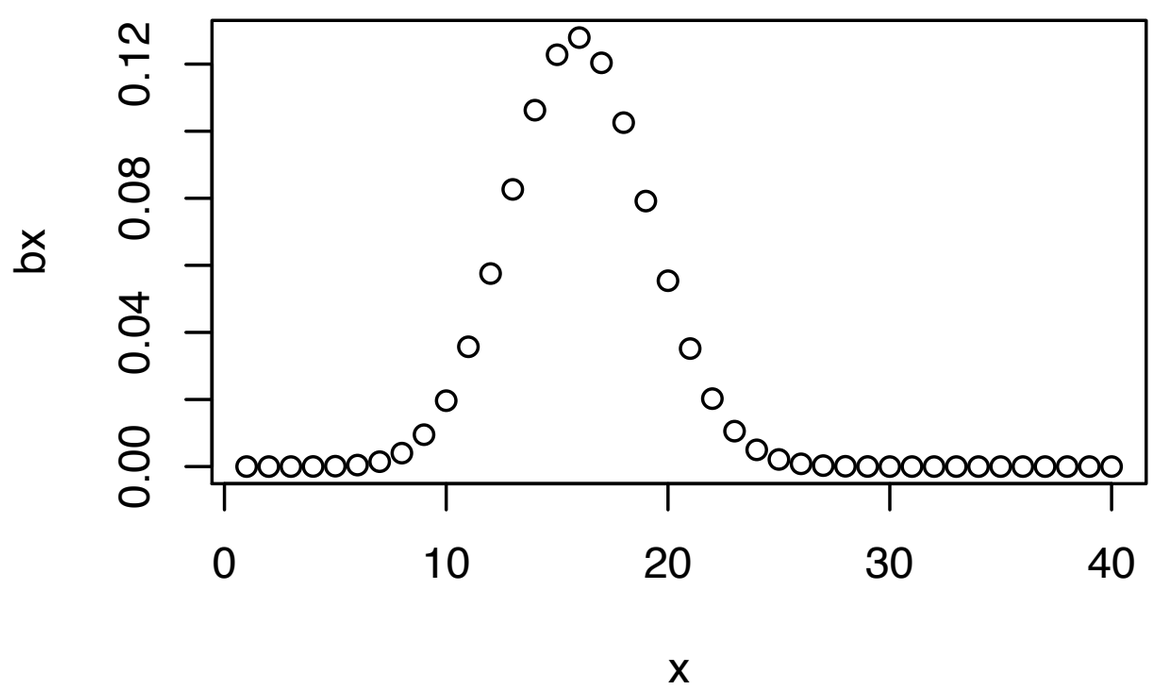
10
15
20
x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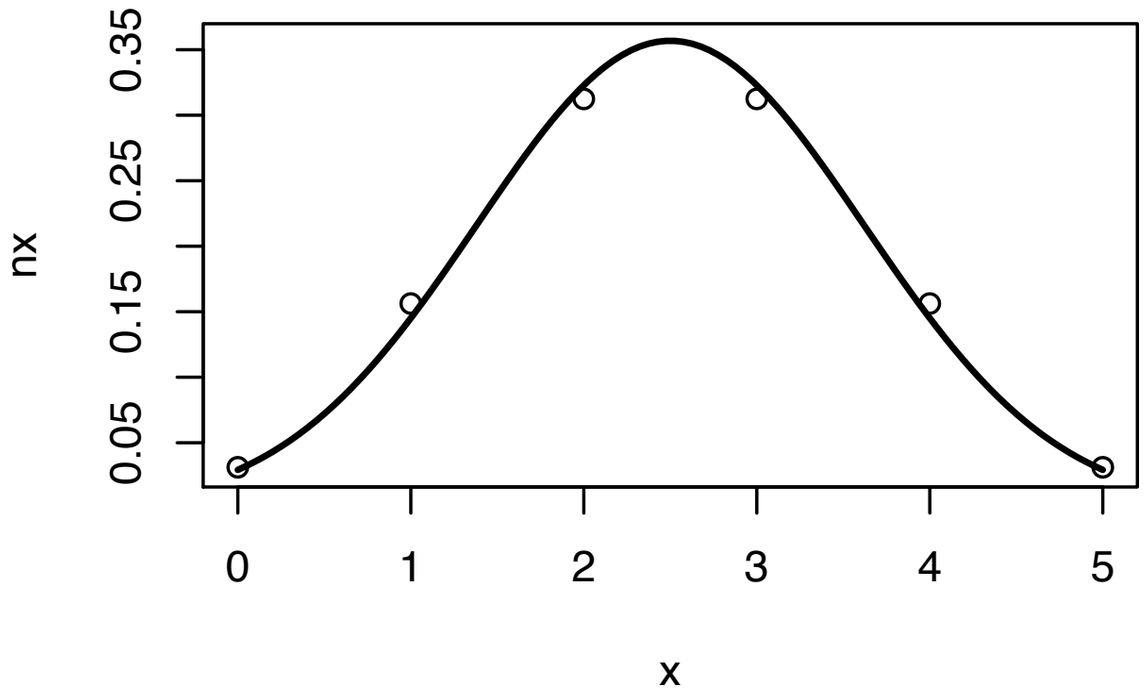
30
40
x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0.25
0.35
0
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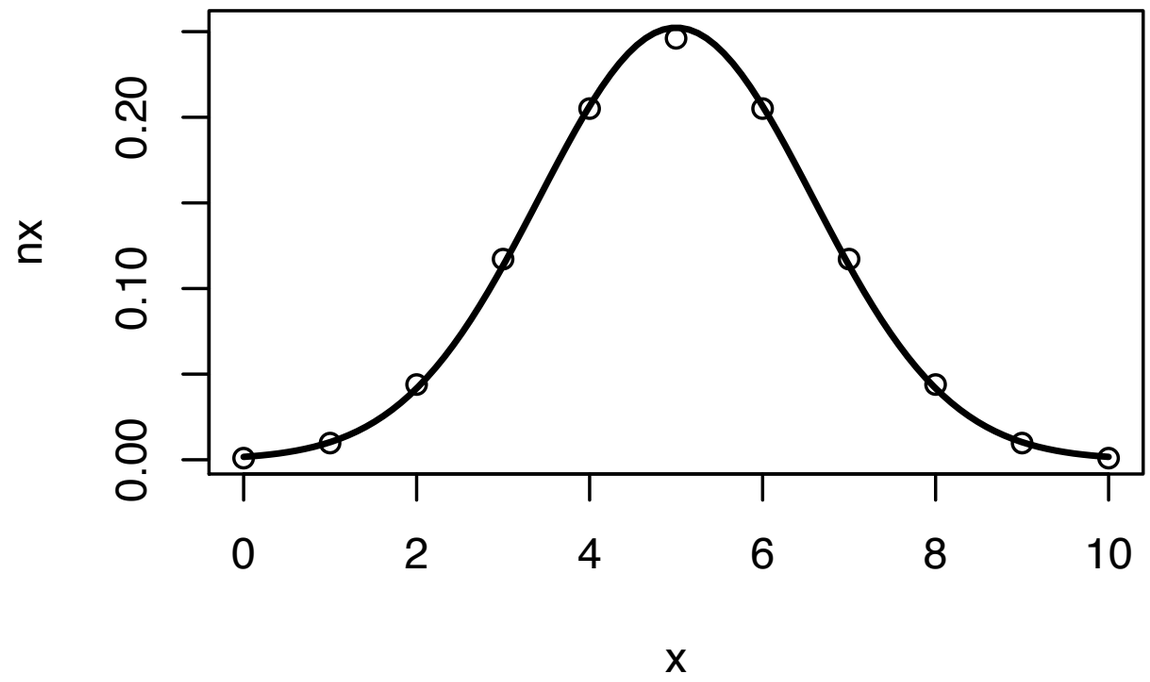
2
3
4
5
x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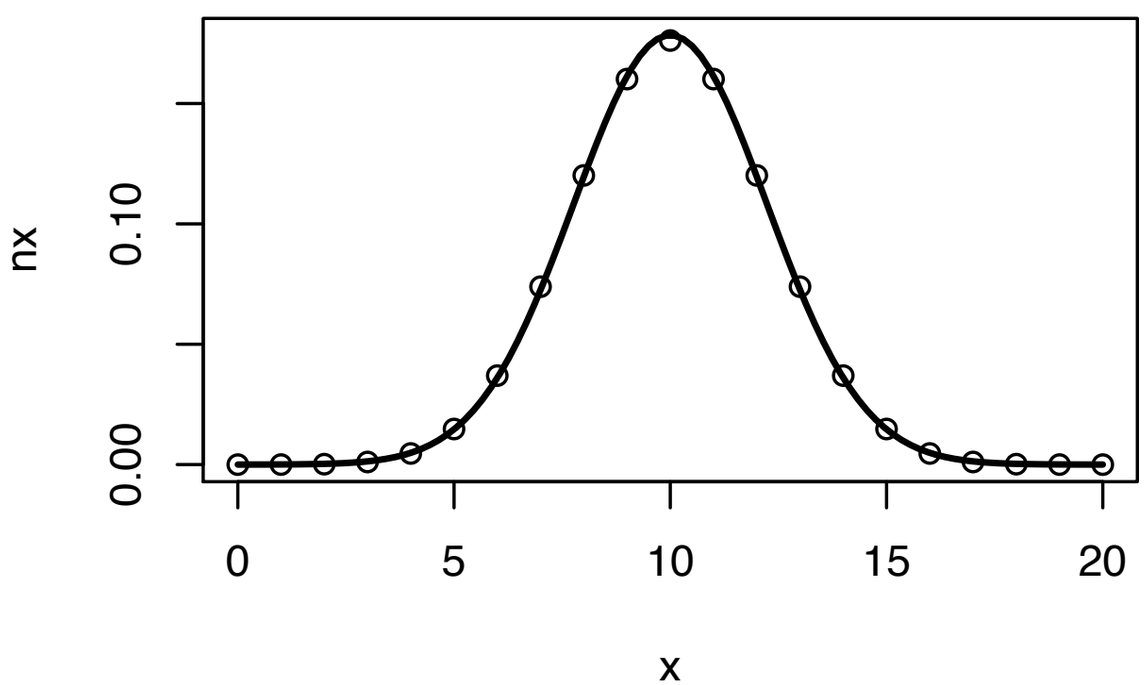
4
6
8
10
x

Coin Flip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20
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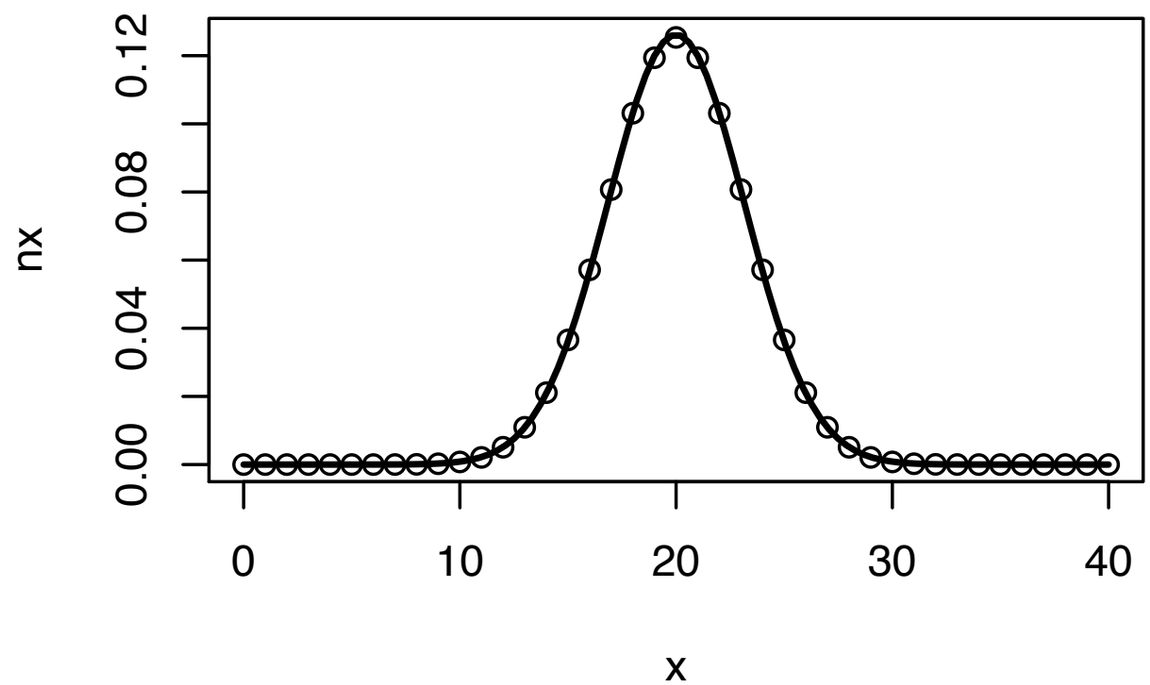
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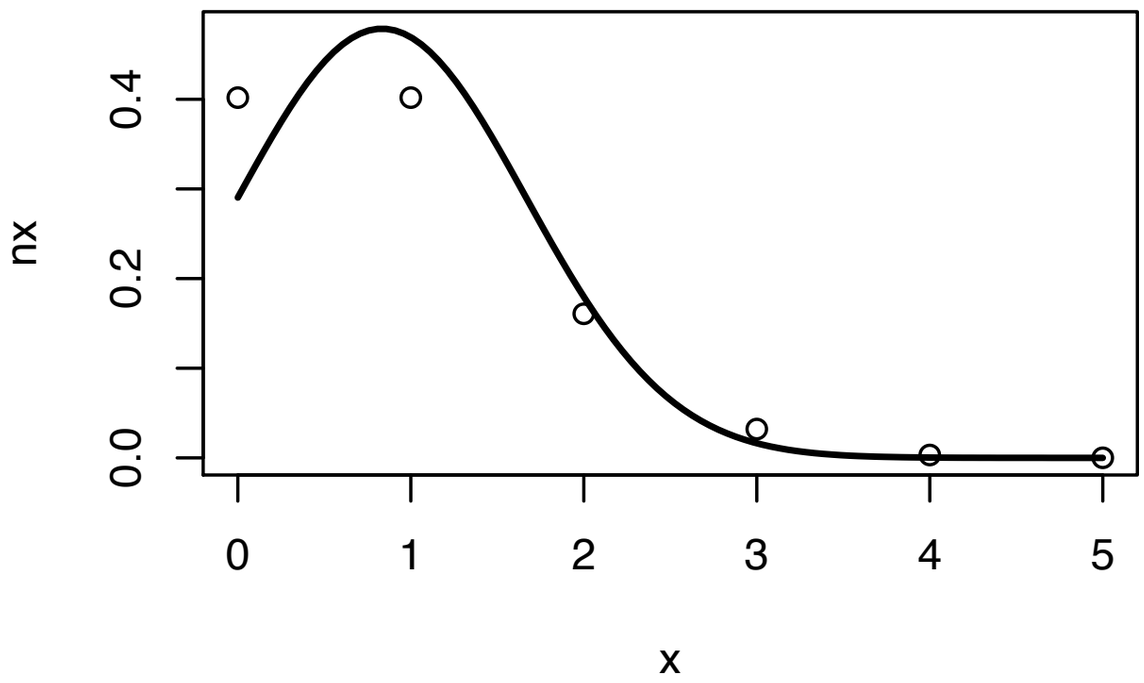
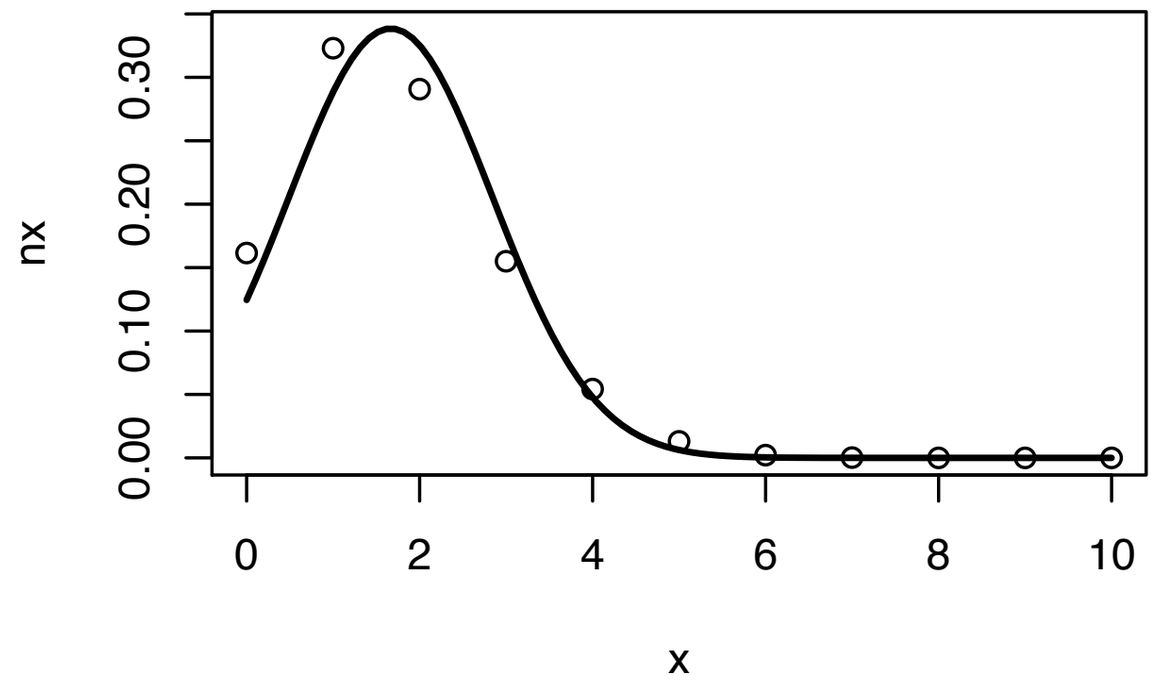
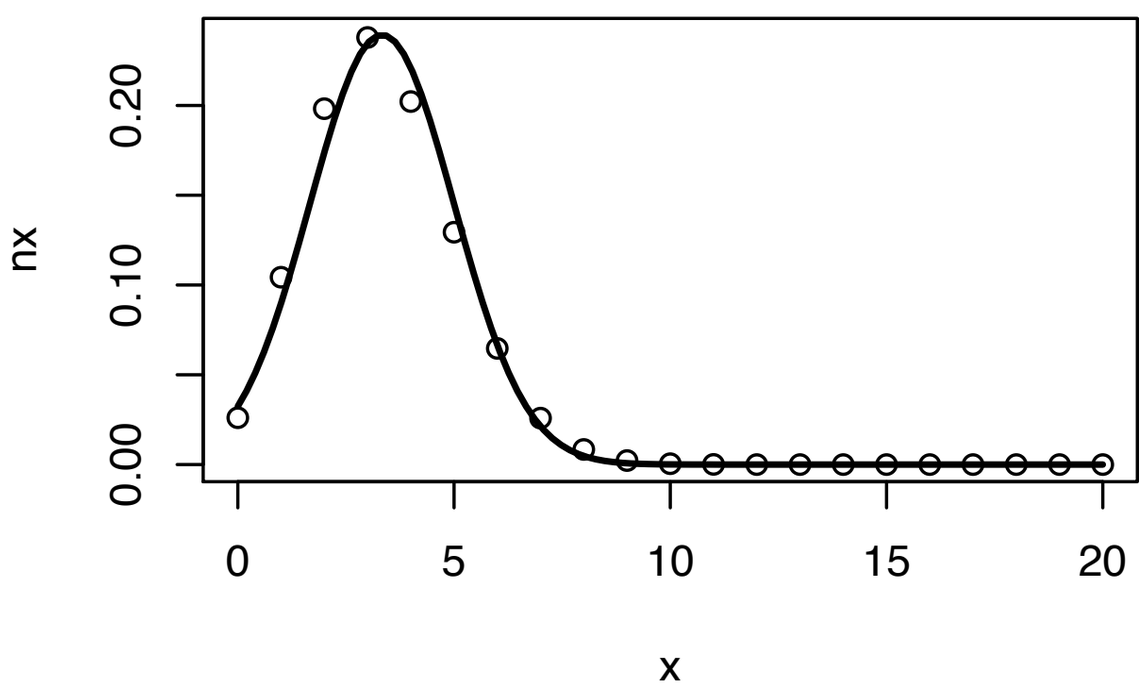
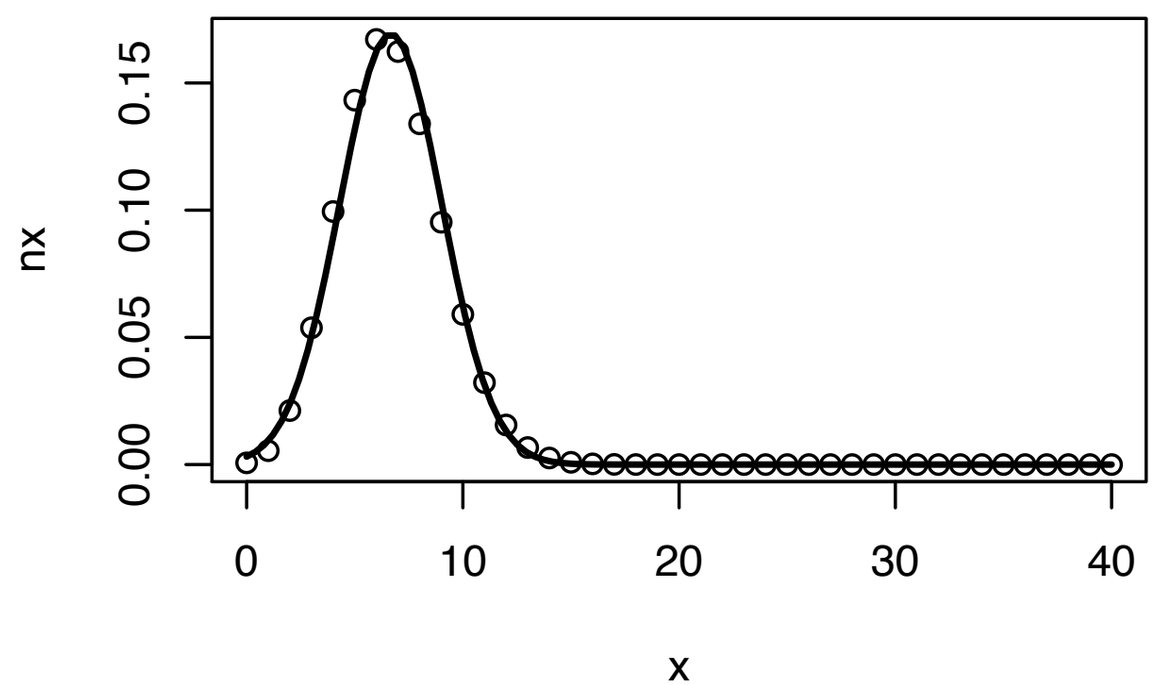
30
40
x





Rolling a Dice





Regression

Regression

- Regression analysis is the most common form of quantitative sociology
- Regression is fundamentally about correlation

Correlation

- For two interval/ratio variables, what happens to one as the other goes up?
- For example, in a cross-sectional sample, as education goes up, what happens to income?

Correlation

- Two variables can be positively or negatively correlated
- Positive - as one increases, so does the other
  - Education and Income
- Negative - as one increases, the other decreases
  - Education and Conservatism

Regression

- Regression measures the amount of correlation, controlling for other variables
- One dependent variable, one or more independent variables

Regression

- Coefficients are the effect of an independent variable on the dependent variable
- A one unit increase in the independent variable predicts a coefficient increase in the dependent variable

Dependent variable: # Children

Variable	Coefficient
Age	0.0332
Education	-0.103
Constant	1.754

Source: GSS

- So for every one year increase in age, the predicted number of children increases by 0.0332, controlling for education

- For every one year increase in education, the predicted number of children decreases by 0.103, controlling for age

Constant

- The constant indicates the predicted value of the dependent variable if all the independent variables were equal to zero
- Often not directly interpretable, because you don't observe a case with values of zero for all IVs
- In previous example, for someone 0 years old, with no education, predicted to have 1.75 children

TABLE 4. Unstandardized Coefficients for the Regression of (ln) Minimum Sentence Length on Explanatory Variables

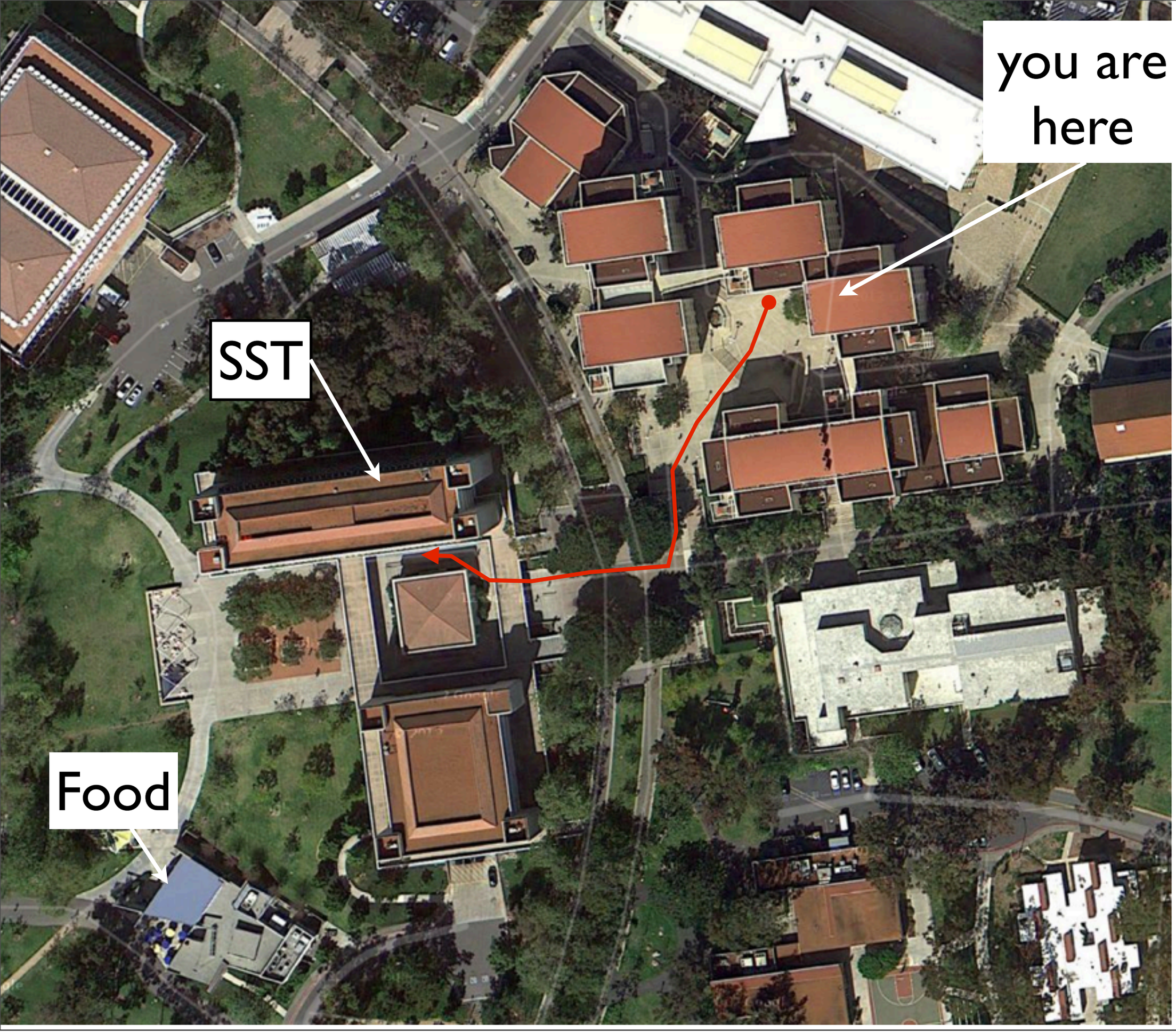
	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6		Model 7	
	b	SE	b	SE	B	SE	b	SE	b	SE	b	SE	B	SE
Intercept	3.195***	.076	3.070***	.094	3.097***	.104	3.094***	.108	3.062***	.120	3.042***	.167	2.871***	.187
3rd degree	1.169***	.063	1.164***	.066	1.162***	.067	1.164***	.067	1.181***	.070	1.174***	.071	1.185***	.071
Vol MS	.623***	.071	.603***	.072	.603***	.073	.606***	.074	.619***	.077	.611***	.078	.620***	.078
Firearm	.144***	.045	.127***	.045	.129**	.047	.132**	.049	.128**	.052	.133**	.053	.135**	.053
# of charges	.005	.004	.004	.004	.003	.004	.003	.004	.002	.004	.003	.004	.002	.004
County jail[a]	−1.142***	.079	−1.117***	.079	−1.117 ***	.079	−1.120***	.079	−1.118***	.080	−1.125	.081	−1.117***	.081
Bench trial			.100*	.044	.098*	.044	.099*	.044	.098*	.080	.099*	.045	.097*	.045
Jury trial			.084	.058	.084	.058	.083	.058	.087	.044	.080	.059	.071	.059
Detained			.102	.059	.109	.061	.110	.061	.117	.058	.045	.146	.313	.196
Pub defender			−.051	.059	−.049	.060	−.051	.061	−.053	.062	−.052	.061	−.057	.061
Ct appt atty			.008	.049	.011	.050	.011	.050	.017	.050	.020	.050	.012	.050
Off <25 yo					.001	.043	.001	.044	.006	.045	.116	.142	.413*	.203
Black off					−.043	.067	−.047	.086	−.033	.087	−.013	.157	.206	.197
Hispanic off					.017	.086	.035	.101	.045	.103	.032	.228	.383	.291
Fem victim							.013	.054	.011	.060	.010	.060	.016	.060
Black victim							.001	.081	.006	.081	.022	.083	.007	.083
Hisp victim							−.034	.090	−.029	.091	−.014	.093	−.020	.093
Intimate partner									.055	.099	.041	.100	.030	.100
Relative									−.080	.076	−.076	.077	−.074	.077
Drunk driver									.082	.100	.079	.102	.054	.102
<25*detain											.019	.117	−.444	.254
Black* <25											−.146	.135	−.510*	.242
Hisp* <25											−.187	.189	−.782*	.369
Black*detain											.074	.145	−.237	.219
Hisp*detain											.153	.209	.347	.330
>25*Black*detain													.540†	.289
>25*Hisp*detain													.836*	.426

* $p \leq .05$, ** $p \leq .01$, *** $p \leq .001$. † $p \leq .06$.

[a]Control variable.

For Friday

- Meeting in SST 155, 9am - noon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